

- 1.21 a) 6 b) -3 c) 2 d) 10 e) 2 f) 81 g) 2 h) $(a+2)^4$
 i) $10^{0,4}$ j) ≈ 1 k) 100 l) $\approx 0,02$ m) $\frac{11}{10a}$ n) 1 o) $x \in \mathbb{R}$ p) -1; 2
 q) $\approx -6,6$ r) 10 s) $n+m$ t) 0 u) 6 v) 1 w) -4 x) $-\frac{8}{3}$

- 1.22 a) $\lg \frac{5}{a}$ b) $\ln \frac{ac}{b}$ c) $\ln \frac{x^2}{y^3(x+y)^5}$ d) $\lg \frac{c^2}{b}$ e) $\ln \sqrt{a-b}$
 f) $\lg \left(\frac{a}{b} \right)^{\frac{2}{3}}$ g) $\ln \frac{\sqrt{a^2+b^2}}{\sqrt[3]{a^2-b^2}}$ h) $\ln 8$ i) $\lg \frac{1}{a^3 \sqrt[3]{b}}$ j) $\lg \sqrt[4]{a^2-b^2}$

1.23 $n \approx 12$ Jahre

- 1.24 a) $e^{x \cdot \ln 3}$ b) $e^{x \ln 2 + 2 \ln x}$ c) $e^{x \ln x}$

- 1.25 a) $t = -T \ln \frac{I}{I_0}$ b) $y = \frac{\ln \varphi}{\ln T_1 - \ln T_2}$
 c) $r_2 = r_1 \cdot e^{\frac{u}{rE}}$ d) $U_x = U_0 \cdot 10^{\frac{ps}{20}}$

1.26 $n \approx 30$

- 2.1 a) $\frac{1}{19}$ b) $x = 1$ falls $a \neq 1$, $x \neq 0$ beliebig, falls $a = 1$

$$c) x = \begin{cases} \frac{a+b}{a-b-1} & \text{falls } a-b-1 \neq 0 \\ \text{beliebig} & \text{falls } a = \frac{1}{2} \text{ und } b = -\frac{1}{2} \\ \text{keine L\"osung} & \text{falls } a-b-1 = 0 \text{ und } a+b \neq 0 \end{cases}$$

d) $x = 12$

e) $x = a - b$, falls $a \neq 2b$

2.2 a) $v_0 = \frac{1}{t} (s + \frac{1}{2}gt^2)$ g) $= 2 \frac{v_0 t - s}{t^2}$

b) $s_1 = s_2 - v(t_2 - t_1)$ $t_1 = \frac{vt_2 + s_1 - s_2}{v}$

c) $h_1 = h_2 - \frac{E_{pot}}{mg}$ $h_1 = h_2 + \frac{E_{pot}}{mg}$

d) $R = \frac{CR_1}{C + 4\pi KR_1}$ $R_1 = \frac{CR}{C - 4\pi KR}$

e) $\Delta t = \frac{m_1 - m_2}{m_2 \lambda_2 - m_1 \lambda_1}$ $\lambda_2 = \frac{m_1(1 + \lambda_1 \Delta t) - m_2}{m_2 \Delta t}$

2.3 65,45min

2.4 38s

2.5 $20k\Omega, 40k\Omega, 120k\Omega$

- 2.6 a) $x \leq \frac{1}{19}$ b) $a > 1 \Rightarrow L = (0, 1)$ f\"ur $a = 1$ keine L\"osung
 $a < 1 \Rightarrow L = \mathbb{R} \setminus [0, 1]$

$a - b < 1 \Rightarrow x \leq a + 2b$

c) $a - b > 1 \Rightarrow x \geq a + 2b$ d) $L = (-\infty, -\frac{3}{2}) \cup [12, \infty)$

$a - b = 1 \wedge b \leq -\frac{1}{3} \Rightarrow x \in \mathbb{R}$

- 2.7 a) $x_1 = 2, x_2 = -7$ b) $x_1 = \frac{1}{2}, x_2 = -1$

c) $x_1 = \frac{3}{2}, x_2 = \frac{3}{4}$ d) $x_1 = 6, x_2 = -\frac{16}{3}$

e) $x_1 = -\frac{40}{11}, x_2 = 4$

- 2.8 a) $x_1 = -\frac{b}{a}, x_2 = \frac{b}{a}$ f\"ur $a \neq 0$; x beliebig f\"ur $a = b = 0$

b) $x_1 = 0, x_2 = R^2$

c) $x_1 = 0, x_2 = \frac{2}{a+b}$ f\"ur $|a| \neq |b|$; x bel. f\"ur $a = b$; $x = 0$ f\"ur $a = -b \neq 0$

d) $x_1 = a - 2b$; $x_2 = b - 2a$ f\"ur $a \neq b$; f\"ur $a = b$ keine L\"osung

2.9 a) $x_1 = \sqrt{7}, x_2 = -\sqrt{7}$ b) $x_1 = -4, x_2 = 4$

c) $x_1 = 81, x_2 = 16$ d) $x_1 = -\frac{1}{5}; x_2 = 2$

2.10 a) $\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$ b) $(x - 2)^2 - 8$ c) $\left(x + \frac{9}{2}\right)^2 - \frac{69}{4}$

2.11 a) $x_{1/2} = -1 \pm \sqrt{6}$ b) $x_{1/2} = 2 \pm \sqrt{20}$ c) $x_1 = \frac{3}{4}, x_2 = \frac{3}{2}$
 d) $x_1 = -4, x_2 = 10$ e) $x_1 = -4, x_2 = -2$ f) $x_{1/2} = -\frac{9}{2} \pm \sqrt{\frac{5}{4}}$
 g) $x_{1/2} = \frac{11}{4} \pm \sqrt{\frac{129}{16}}$ h) $x \notin \mathbb{R}$ i) $x_1 = \frac{1}{2}, x_2 = \frac{6}{5}$

2.12 151m

2.13 26, 8Ω; 226, 8Ω

2.14 0,67mm

2.15 4h

2.16 a) $L = [-1 - \sqrt{6}, -1 + \sqrt{6}]$ b) $L = (-\infty, 2 - \sqrt{20}) \cup (2 + \sqrt{20}, \infty)$
 c) $L = \emptyset$ d) $L = (-\infty, \frac{3}{4}] \cup [\frac{3}{2}, \infty)$

2.17 0

2.18 a) $-\frac{1}{2}$ b) $\{-6; 2\}$ c) $\{\frac{1}{4}\}$ d) $\{-8; \frac{1}{4}\}$
 e) $L = (-\infty, -\frac{1}{2})$ f) $L = [-2, 6]$ g) $L = (-\infty, \frac{1}{4}]$ h) $L = (-\infty, -8) \cup (\frac{1}{4}, \infty)$

2.19 a) $x_1 = -2; x_2 = 8$ b) $x_1 = -7; x_2 = -\frac{1}{3}$

c) $(-1, 5)$ d) $(-\infty, \frac{1}{5}) \cup (9, \infty)$
 e) $[-5, 3]$ f) $(-\infty, -2 - \sqrt{11}) \cup (-2 - \sqrt{7}, -2 + \sqrt{7}) \cup (-2 + \sqrt{11}, \infty)$

2.20 a) $(-\infty, -\frac{1}{2})$ b) $[-6, 2]$ c) $(-\infty, \frac{1}{4}]$
 d) $(-\infty, -8) \cup (-\frac{5}{3}, \frac{1}{4})$ e) $(-\infty, -4] \cup [4, \infty)$ f) $(-\infty, 2) \cup (\frac{23}{11}, \infty)$

2.21 a) \emptyset b) $\frac{23}{5}$ c) 17 d) 49 e) $\frac{1}{4}$

2.22 Falsch: $\sqrt{(a-1)^2} = a-1$ Richtig: $\sqrt{(a-1)^2} = |a-1|$

3.1 a) $l = \sqrt{26} = 5.099; \alpha = -78,96^\circ$ b) $l = 2\sqrt{13} = 7.211; \alpha = -56,31^\circ$

3.2 a) $y = \frac{\sqrt{3}}{3}(x+2) - 1 = 0.57735x + 0.1547$

b) $y = -\sqrt{3}(x+1) - 2 = -1.321x - 3.7321$

c) $y = -x$

d) $y = 1.5 + (\sqrt{2} - 1)(x + 0.5) = 1.7071 + 0.41421x$

3.3 a) $y = \frac{4}{3}x - \frac{4}{3}$ b) $y = -\frac{5}{3}x + \frac{1}{3}$ c) $y = -2x + 0,8$ d) $y = x - a + 2$

3.4 a) $y = 2x - 4$ b) $y = -\frac{5}{4}x + \frac{23}{4}$

3.5 $y = \frac{\sqrt{3}}{3}x + 1 - \frac{\sqrt{3}}{3}$

3.6 a) $r = 1, M = (-2; 2)$ b) $r = 3, M = (3; -1)$

3.7 $x^2 + y^2 + 4x - 6y - 12 = 0$

3.8 $(x-2, 1)^2 + (y-2, 1)^2 = 4,41$ bzw. $(x-11, 9)^2 + (y-11, 9)^2 = 141,6$

3.9 a) $\frac{\pi}{4}$ b) $\frac{3\pi}{4}$ c) $\frac{4\pi}{3}$ d) $\frac{7\pi}{4}$ e) 120° f) 135° g) 330° h) 210°

3.10 a) 0,624 b) 3,804 c) $143^\circ 48' 45''$ d) $49^\circ 50' 50''$

3.12	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{4\pi}{3}$	$\frac{7\pi}{4}$	120°	135°	330°	210°	270°
$\sin x$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
$\cos x$	$\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}$	$-\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}\sqrt{3}$	0
$\tan x$	1	-1	$\sqrt{3}$	-1	$-\sqrt{3}$	-1	$-\frac{1}{3}\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	$n.d.$
$\cot x$	1	-1	$\frac{1}{3}\sqrt{3}$	-1	$-\frac{1}{3}\sqrt{3}$	-1	$-\sqrt{3}$	$\sqrt{3}$	0

3.13 a) $\cos \alpha = \pm \frac{7}{25}$ $\tan \alpha = \pm \frac{24}{7}$ $\cot \alpha = \pm \frac{7}{24}$

b) $\sin \alpha = \pm \frac{1-n^2}{1+n^2}$ $\tan \alpha = \pm \frac{1-n^2}{2n}$ $\cot \alpha = \pm \frac{2n}{1-n^2}$

c) $\cot \alpha = \frac{5}{12}$ $\sin \alpha = \pm \frac{12}{13}$ $\cos \alpha = \pm \frac{5}{13}$

3.14 a) $-\sin \frac{3}{2}x$ b) $-\sin \frac{1}{2}x$ c) $-\tan x$ d) $\sin x$ e) $\cos x$

3.15 a) $x_1 = 120^\circ + k \cdot 720^\circ = \frac{2\pi}{3} + 4k\pi$, $x_2 = 240^\circ + k \cdot 720^\circ = \frac{4\pi}{3} + 4k\pi$ ($k \in \mathbb{Z}$)

b) $x_1 = \frac{\pi}{18} + \frac{2k\pi}{3}$, $x_2 = \frac{5\pi}{9} + \frac{2k\pi}{3}$ ($k \in \mathbb{Z}$) c) $x = 157,5^\circ + k \cdot 180^\circ$ ($k \in \mathbb{Z}$)

3.16 313,26 m

3.17 $\gamma = 41,73^\circ$

3.18 $F_R = 657 \text{ N}$; $\varepsilon = 33^\circ 20'$

4.1 Def.-Bereich bei a) - d) sowie i) - k): jeweils \mathbb{R} , e) jeweils $\mathbb{R} \setminus \{-1\}$,

f) $[-1, \infty)$, $[-1, \infty)$, $(-\infty, 1]$, h) $(1, \infty)$, $(1, \infty)$, $(-\infty, 1)$,

4.2 Falls es mehrere Möglichkeiten gibt, geben wir nur eine an:

a) $D_f = [0, \infty)$, $y = f^{-1}(x) = +\sqrt{x-1}$ b) $x \geq -2$, $y = f^{-1}(x) = \frac{x}{2} - 2$

c) $y = f^{-1}(x) = \sqrt[3]{\frac{x-2}{3}}$ d) $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$, $y = f^{-1}(x) = \frac{1}{2} \arcsin x$

e) $-\frac{3}{2} < x < \infty$, $y = f^{-1}(x) = \frac{1}{2}(e^{x+4} - 3)$

4.3 a) $y = 2e^{(\ln 3) \cdot 2x}$ b) $y = e^{(\ln 0,3) \cdot 4x} - 1$ c) $y = \frac{3}{\ln 5} \ln x$ d) $y = \frac{2}{\ln 10} \ln(3x + 1)$

4.4 Der Graph von a) wird verschoben um

b) y_0 nach oben c) x_0 nach rechts d) x_0 nach rechts und y_0 nach oben

4.6 a) 2π b) π c) $\frac{\pi}{2}$ 4.7 ca. 165°

4.8 a) $y = \frac{5}{2}x - \frac{1}{2}$ b) $y = 4x^2 - 1$ 4.9 a) $63,4^\circ$ b) $71,6^\circ$ c) 76°

5.1 a) $y' = -2x^3 + 2x^2 + \frac{1}{x^2}$ $x \neq 0$ b) $y' = 3a^2x^2 - 2\sqrt{b}x + \frac{c}{2}$

c) $y' = 1$ d) $y' = \frac{5}{x^6} - \frac{6}{x^4} - \frac{2}{x^3}$ $x \neq 0$

e) $y' = \frac{\cos x (\sin^2 x + 2)}{(1 + \cos^2 x)^2}$ f) $y' = \arcsin x + \frac{x}{\sqrt{1-x^2}}$ $|x| < 1$

g) $y' = \frac{1}{6}x^{-5/6}$ $x \neq 0$ h) $y' = \frac{6}{x^3} + 2$ $x \neq 0$

i) $y' = \frac{1-\ln x}{x^2}$ $x > 0$ j) $y' = -8x^3(1-x^4)$

k) $y' = -800x^3(1-x^4)^{199}$ l) $y' = \cos x \cdot 4^x + \ln 4 \cdot 4^x \sin x$

m) $y' = 2 \ln 3 \cdot x \cdot 3^{x^2}$ n) $y' = 2 \ln 3 \cdot (3^x)^2$

o) $y' = -\frac{2a}{x^2} \cos\left(\frac{a}{x}\right)$ $x \neq 0$ p) $y' = \frac{1}{1+x^2}$ $|x| \neq 1$

q) $y' = \frac{2x}{3\sqrt[3]{(x^2-1)^2}}$ $|x| \neq 1$ r) $y' = e^x \tan x + \frac{e^x}{\cos^2 x}$

s) $y' = \frac{x}{\sqrt{1+x^2}(1+\sqrt{1+x^2})}$ t) $y' = -\frac{4}{(e^x+e^{-x})^2}$ $x \neq 0$ u) $y' = (1-2x^2)e^{-x^2}$

- 5.2 a) $\frac{dR}{d\omega} = -At \sin(x + \omega t)$ b) $\frac{dv}{dR_1} = -\frac{1}{R_1^2}$
c) $\frac{ds}{dt} = -gt + a \cos(\alpha t)$ d) $\frac{dA}{dB} = C \cdot \ln D \cdot D^B - F \cdot D \cdot B^{D-1}$
e) $\frac{\partial \varphi}{\partial t} = \omega_0 \frac{1}{(k\omega_0 t + 1)}$ f) $\frac{dy}{ds} = \frac{g}{v} e^{-\frac{2ks}{m}}$
g) $\frac{dR}{dT} = -\frac{a \cdot b}{T^2} e^{\frac{b}{T}}$ h) $\frac{dl}{dn} = zU_q \frac{zR_a - R_i n^2}{(zR_a + R_i n^2)^2}$
i) $\frac{dA}{d\omega} = -2 \frac{\omega F_0}{m} \frac{\omega^2 - \omega_0^2 + 2\delta^2}{\sqrt{[(\omega - \omega_0)^2 + 4\delta^2 \omega^2]^3}}$

- 5.3 a) $\varphi = 26,6^\circ$ b) $\varphi = 28,4^\circ$ c) $\varphi = 70^\circ$

5.4 In einigen Lösungen (die Reihenfolge in der Tabelle ist wie bei den Aufgaben) kommt der Ausdruck $\operatorname{sgn}(\dots)$ vor. Man verwendet diesen Ausdruck für eine komprimierte Schreibweise, um eine Fallunterscheidung nicht aufzuschreiben zu müssen. Er bedeutet das Vorzeichen:

$$\operatorname{sgn}(T(x)) = \begin{cases} 1, & \text{wenn } T(x) > 0 \\ -1, & \text{wenn } T(x) < 0 \\ 0, & \text{wenn } T(x) = 0 \end{cases} = \frac{|T(x)|}{T(x)}, \text{ wenn } T(x) \neq 0.$$

Diese Aufgaben sind als etwas schwieriger einzustufen. (Für Anfänger nicht geeignet!)

$\frac{-x^2+2x-4}{(x^2-4)^2}$	$-\frac{1}{(x+1)^2}$	$\frac{-x^2-2x+4}{(x^2+4)^2}$
$\frac{-3x^2+4x+4}{(x^2+4)^3}$	$e^x(1+x)$	$\frac{2x}{1+x^2}$
$\frac{(x^3-x)\cos x-(x^2+1)\sin x}{(x^2-1)^2}$	$\frac{2\cos 4x}{\sqrt{\sin 4x}}$	$\frac{5}{2\sqrt{(x-2)(2x+1)^3}}$
$x^{\sin x} \left(\frac{\sin x}{x} + \cos x \cdot \ln x \right)$	$-\frac{1}{(3x-1)^2} + \frac{5x^2-2x}{\sqrt{2x-1}}$	$2 \ln x + (\ln x)^2$
$2x(1-x^2)e^{-x^2} + e^x(\cos x - \sin x)$	$\frac{-2x}{(1+x^2)\sqrt{x^2}} = -\frac{2\operatorname{sgn} x}{1+x^2}$	$-\frac{\cos^2 x}{\sin^3 x}$
$\frac{1+x^2}{1+x^2+x^4}$	$-\frac{1}{\cos x}$	$2x \sin \frac{1}{x} - \cos \frac{1}{x}$
$-\frac{ x }{x^2 \sqrt{x^2-1}}$	$\frac{7}{8\sqrt[8]{x}}$	$\frac{2+\ln x}{2\sqrt{x}} x \sqrt{x}$
$-\frac{1}{x(\ln x)^2}$	$\frac{3}{x(x^2+3)}$	$\frac{4}{3} \cdot \frac{\operatorname{sgn}(2x+1)}{\sqrt[3]{ 2x+1 }}$
$[a \cos(bx) - b \sin(bx)] e^{ax}$	$\frac{2x^3}{1-x^4}$	$\frac{2\operatorname{sgn}(1-x^2)}{1+x^2}$
$\frac{1}{\sqrt{x^2+a^2}}$	$2\operatorname{sgn} x, x \neq 0$	$-\sin(2x) + \cos x \cdot \operatorname{sgn} x$

- 5.5 a) $3x^{\frac{1}{3}} + C$ b) $\frac{\sqrt{2}}{7} x^{\frac{7}{2}} + C$ c) $-\frac{1}{\ln 2} \cdot \left(\frac{1}{2} \right)^x + C$
5.6 a) $\frac{3}{5} \ln|x| + \frac{4}{5} x^{-1} + C$ b) $\frac{4}{3} x^3 + 6x^2 + 9x + C$ c) $2e^x - 7 \sin x + C$
5.7 a) $2x^3 - \frac{1}{2} \cos 2x + C$ b) $\frac{4}{3} x^{3/2} + x + 6\sqrt{x} + C$ c) $3e^x + \sin x + C$
5.8 a) $\frac{x^2}{2} - x + C$ b) $-\frac{1}{5.99 \cdot (2-5x)^{99}} + C$ c) $\frac{1}{3} \tan(3x+3) + C$
5.9 a) $-\frac{\cos^6 x}{6} + C$ b) $\frac{\ln^2 x}{2} + C$ c) $\frac{\ln(x^2+9)}{2} + C$
5.10 a) $-\frac{e^{-x^2}}{2} + C$ b) $\frac{1}{10(1-5x^3)^2} + C$ c) $e^{\tan x} + C$
5.11 a) $\frac{\ln(x^2+10x+30)}{2} + C$ b) $\frac{\sin(x^2+1)}{2} + C$ c) $\ln|\ln x| + C$
5.12 a) $-\frac{1}{2(e^x-3)^2} + C$ b) $\ln(x^2-x+5) + C$ c) $\sqrt{2x-x^2} + C$
5.13a) $\frac{1}{2}(x^2+1-\ln(x^2+1)) + C$ b) $\arctan e^x + C$ c) $-\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$