Aufgabe: Ermitteln Sie von dem gegebenen reellen Signal im Zeitbereich

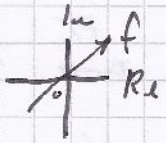
$u(t) = -2V + 4V \cdot \sin\left(2\pi \cdot 10\,Hz \cdot t + \frac{\pi}{4}\right)$

die Fourier-Transformierte als Darstellung

a) von Real- und Imaginärteil

b) Betrag und Phase

c) 3-dimensionale Darstellung (Real- / Imaginärteil und Frequenz)



a) $x(t) = x_0 \cdot \sin(2\pi f_0 t) \overset{FT}{\circ\!\!-\!\!\bullet} \underline{X}(f) = j\frac{x_0}{2}\left[\delta(f+f_0) - \delta(f-f_0)\right]$

$\bar{x} = x_0 \overset{FT}{\circ\!\!-\!\!\bullet} X(f) = x_0 \cdot \delta(f)$

$x_2(t) = x_1(t-t_0) \overset{FT}{\circ\!\!-\!\!\bullet} \underline{X}_2(f) = \underline{X}_1(f) \cdot e^{-j2\pi t_0 f}$

$u_1(t) = -2V \quad \circ\!\!-\!\!\bullet \quad u_1(f) = -2V\,\delta(f)\,Hz^{-1}$

$u_2(t) = 4V \cdot \sin(2\pi \cdot 10\,Hz \cdot t + \frac{\pi}{4})$

$u_{21}(t) = 4V \cdot \sin(2\pi \cdot 10\,Hz \cdot t)$

$u_{21}(f) = j\frac{4V}{2}\left[\delta(\right.$

$$2\pi\ 10\,Hz\,(t-t_0) = 2\pi\ 10\,Hz\,t + \frac{\pi}{4}$$

$$2\pi\ 10\,Hz\,t - 2\pi\ 10\,Hz\,t_0 = 2\pi\ 10\,Hz\,t + \frac{\pi}{4}$$

$$t - 2\pi\ 10\,Hz\,t_0 = t + \frac{\pi}{4}$$

$$t_0 = -\frac{\frac{\pi}{4}}{2\pi\ 10\,Hz} = -\frac{1}{80}\,s$$

$$u_2(t) = 4V \cdot \sin(2\pi\ 10\,Hz\,t)$$

$$\underline{u}_2(f) = j\frac{4V}{2}\left[\delta(f+10\,Hz)\,Hz^{-1} - \delta(f-10\,Hz)\,Hz^{-1}\right]$$

$$u_3(t) = 4V \sin\left[2\pi\ 10\,Hz\,(t+\frac{1}{80}\,s)\right]$$

$$\underline{u}_3(f) = u_2(f) \cdot e^{-j2\pi\left(-\frac{1}{80}s\right)\cdot f}$$

Ausblenden

$$\underline{u}_3(f) = j2V\,\delta(f+10\,Hz)\,Hz^{-1} \cdot e^{j\frac{\pi}{40}f}$$
$$- j2V\,\delta(f-10\,Hz)\,Hz^{-1} \cdot e^{j\frac{\pi}{40}f}$$

Ausblenden

$$\underline{u}_3(f) = j2V \cdot e^{j\frac{\pi}{40}s(-10\,Hz)} \cdot \delta(f+10\,Hz)\,Hz^{-1}$$
$$- j2V \cdot e^{j\frac{\pi}{40}s(10\,Hz)} \cdot \delta(f-10\,Hz)\,Hz^{-1}$$

$$\underline{u}_3(f) = j2V\,e^{-j\frac{\pi}{4}}\,\delta(f+10\,Hz)\,Hz^{-1}$$
$$- j2V\,e^{j\frac{\pi}{4}}\,\delta(f-10\,Hz)\,Hz^{-1}$$

$$e^{-j\frac{\pi}{4}} = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$\underline{u}_3(f) = j2V \cdot \left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) \cdot \delta(f+10\,\mathrm{Hz})\,\mathrm{Hz}^{-1}$$

$$- j2V\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)\delta(f-10\,\mathrm{Hz})\,\mathrm{Hz}^{-1}$$

$$\underline{u}_3(f) = j\sqrt{2}V\delta(f+10\,\mathrm{Hz})\,\mathrm{Hz}^{-1} + \sqrt{2}V\,\delta(f+10\,\mathrm{Hz})\,\mathrm{Hz}^{-1}$$

$$- j\sqrt{2}\,\delta(f-10\,\mathrm{Hz})\,\mathrm{Hz}^{-1} + \sqrt{2}V\,\delta(f-10\,\mathrm{Hz})\,\mathrm{Hz}^{-1}$$

$$\underline{u}_{\mathrm{Gl}}(f) = -2V\,\delta(f)\,\mathrm{Hz}^{-1} \quad (\text{Gleichanteil})$$

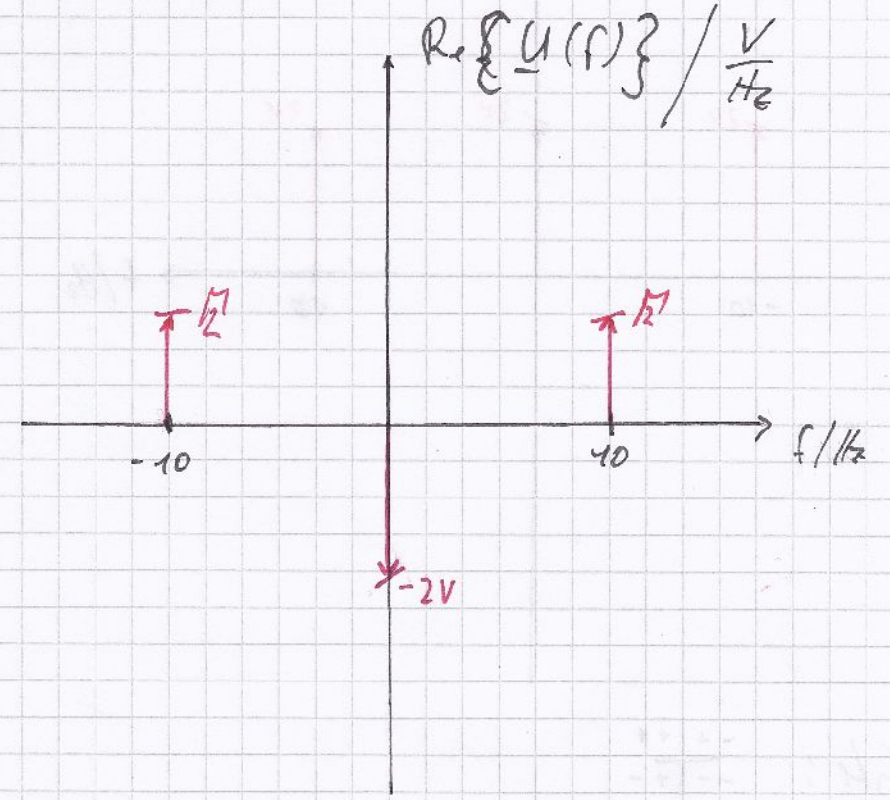
$$\underline{u}(f) = -2V\delta(f)\mathrm{Hz}^{-1} + j\sqrt{2}V\delta(f+10\,\mathrm{Hz})\,\mathrm{Hz}^{-1} + \sqrt{2}V\delta(f+10\,\mathrm{Hz})\,\mathrm{Hz}^{-1}$$

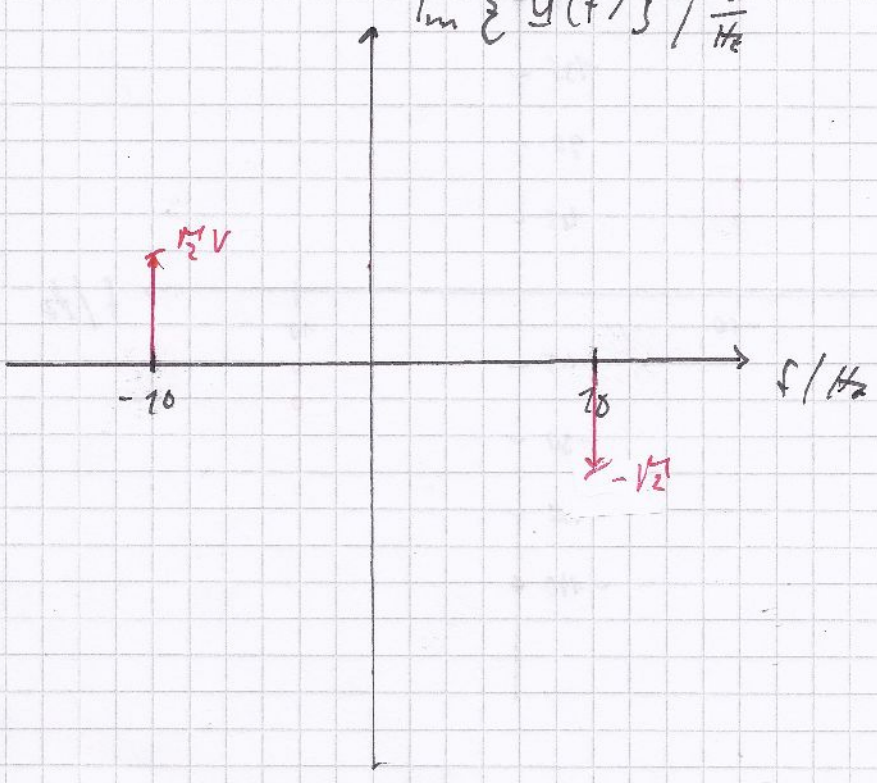
$$- j\sqrt{2}V\delta(f-10\,\mathrm{Hz})\,\mathrm{Hz}^{-1} + \sqrt{2}V\,\delta(f-10\,\mathrm{Hz})\,\mathrm{Hz}^{-1}$$

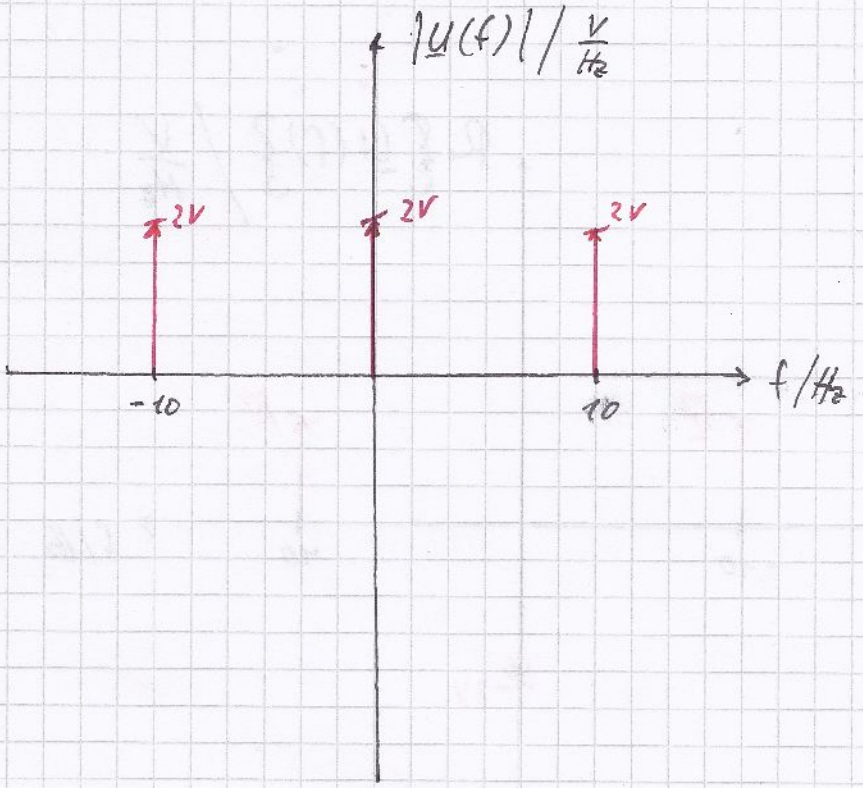
b)

$$\mathrm{Re}\{\underline{u}(f)\} = -2V\,\delta(f)\,\mathrm{Hz}^{-1} + \sqrt{2}\,\delta(f+10\,\mathrm{Hz})\,\mathrm{Hz}^{-1} + \sqrt{2}V\,\delta(f-10\,\mathrm{Hz})\,\mathrm{Hz}^{-1}$$

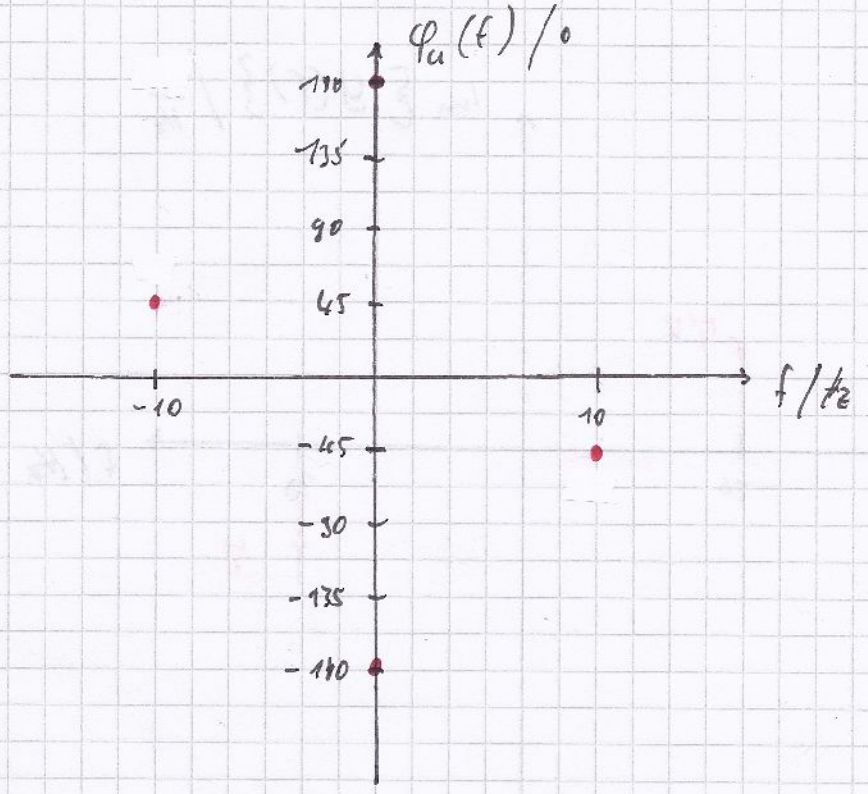
$$\mathrm{Im}\{\underline{u}(f)\} = j\sqrt{2}V\,\delta(f+10\,\mathrm{Hz})\,\mathrm{Hz}^{-1} - j\sqrt{2}\,\delta(f-10\,\mathrm{Hz})\,\mathrm{Hz}^{-1}$$


Re{U(f)} / V/Hz
√2
√2
-10
10
f/Hz
-2V


Im{U(f)} / V/Hz
√2V
-10
10
f/Hz
-√2



Beachte: $\begin{array}{c|c} - + & + + \\ \hline - - & + - \end{array}$



c)