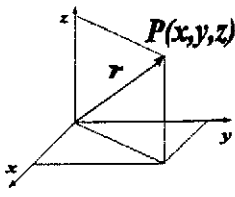
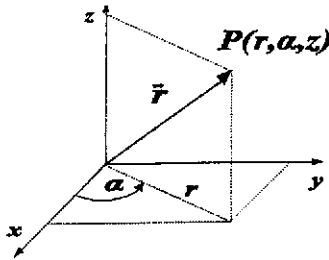
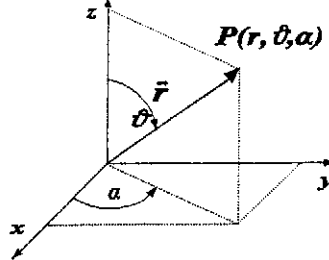


**Tabelle 1:** Darstellung des Ortsvektors, der Tangenteneinheitsvektoren und Lamé-Koeffizienten in den gebräuchlichen orthogonalen Koordinatensystemen

	kartesisch	zylindrisch	sphärisch
$r$	$\begin{aligned} r &= x e_x \\ &+ y e_y \\ &+ z e_z \end{aligned}$	$\begin{aligned} r &= r \cos\alpha e_x \\ &+ r \sin\alpha e_y \\ &+ z e_z \end{aligned}$	$\begin{aligned} r &= r \cos\alpha \sin\vartheta e_x \\ &+ r \sin\alpha \sin\vartheta e_y \\ &+ r \cos\vartheta e_z \end{aligned}$
			
$\frac{\partial r}{\partial x_i}$	$\begin{aligned} \frac{\partial r}{\partial x} &= e_x \\ \frac{\partial r}{\partial y} &= e_y \\ \frac{\partial r}{\partial z} &= e_z \end{aligned}$	$\begin{aligned} \frac{\partial r}{\partial r} &= \cos\alpha e_x \\ &+ \sin\alpha e_y \\ \frac{\partial r}{\partial \alpha} &= -r \sin\alpha e_x \\ &+ r \cos\alpha e_y \\ \frac{\partial r}{\partial z} &= e_z \end{aligned}$	$\begin{aligned} \frac{\partial r}{\partial r} &= \cos\alpha \sin\vartheta e_x \\ &+ \sin\alpha \sin\vartheta e_y \\ &+ \cos\vartheta e_z \\ \frac{\partial r}{\partial \vartheta} &= r \cos\alpha \cos\vartheta e_x \\ &+ r \sin\alpha \cos\vartheta e_y \\ &- r \sin\vartheta e_z \\ \frac{\partial r}{\partial \alpha} &= -r \sin\alpha \sin\vartheta e_x \\ &+ r \cos\alpha \sin\vartheta e_y \end{aligned}$
$h_i$	$\begin{aligned} h_1 &= h_x = 1 \\ h_2 &= h_y = 1 \\ h_3 &= h_z = 1 \end{aligned}$	$\begin{aligned} h_1 &= h_r = 1 \\ h_2 &= h_\alpha = r \\ h_3 &= h_z = 1 \end{aligned}$	$\begin{aligned} h_1 &= h_r = 1 \\ h_2 &= h_\vartheta = r \\ h_3 &= h_\alpha = r \sin\vartheta \end{aligned}$
$e_i$	$\begin{aligned} e_1 &= e_x \\ e_2 &= e_y \\ e_3 &= e_z \end{aligned}$	$\begin{aligned} e_1 &= e_r = \cos\alpha e_x \\ &+ \sin\alpha e_y \\ e_2 &= e_\alpha = -\sin\alpha e_x \\ &+ \cos\alpha e_y \\ e_3 &= e_z = e_z \end{aligned}$	$\begin{aligned} e_1 &= e_r = \frac{\partial r}{\partial r} \\ e_2 &= e_\vartheta = \frac{1}{r} \frac{\partial r}{\partial \vartheta} \\ e_3 &= e_\alpha = \frac{1}{r \sin\vartheta} \frac{\partial r}{\partial \alpha} \end{aligned}$

**Tabelle 2: Die Differentialoperatoren in allgemeinen Koordinaten**

	Definition	Darstellung in allgemeinen, krummlinigen Koordinaten
$\text{grad } \varphi$ $(\nabla\varphi)$	$\text{grad } \varphi = \lim_{\Delta V \rightarrow 0} \frac{\oint \varphi dA}{\Delta V}$	$\text{grad } \varphi = \sum_{i=1}^3 e_i \frac{1}{h_i} \frac{\partial \varphi}{\partial x_i}$
$\text{div } \mathbf{v}$ $(\nabla \cdot \mathbf{v})$	$\text{div } \mathbf{v} = \lim_{\Delta V \rightarrow 0} \frac{\oint \mathbf{v} \cdot d\mathbf{A}}{\Delta V}$	$\text{div } \mathbf{v} = \frac{1}{h} \sum_{i=1}^3 \frac{\partial}{\partial x_i} \left( \frac{h}{h_i} e_i \cdot \mathbf{v} \right)$
$\text{rot } \mathbf{v}$ $(\nabla \times \mathbf{v})$	$\text{rot } \mathbf{v} = \lim_{\Delta V \rightarrow 0} \frac{\oint d\mathbf{A} \times \mathbf{v}}{\Delta V}$	$\text{rot } \mathbf{v} = \frac{1}{h} \sum_{i=1}^3 \frac{\partial}{\partial x_i} \left( \frac{h}{h_i} e_i \times \mathbf{v} \right)$ $= \frac{1}{h} \begin{vmatrix} e_1 h_1 & e_2 h_2 & e_3 h_3 \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ v_1 h_1 & v_2 h_2 & v_3 h_3 \end{vmatrix}$
$\Delta \varphi$ $(\nabla^2 \varphi)$	$\text{div grad } \varphi$	$\Delta \varphi = \frac{1}{h} \sum_{i=1}^3 \frac{\partial}{\partial x_i} \left( \frac{h}{h_i^2} \frac{\partial \varphi}{\partial x_i} \right)$
$\Delta \mathbf{v}$	$\text{grad div } \mathbf{v} - \text{rot rot } \mathbf{v}$ $= \nabla(\nabla \cdot \mathbf{v}) - \nabla \times (\nabla \times \mathbf{v})$	siehe z.B.: Wunsch/Schulz: <i>Elektromagnetische Felder.</i> Verlag Technik, Berlin, 1989, S. 89

**Tabelle 3:** Die Differentialoperatoren in den gebräuchlichsten orthogonalen Koordinatensystemen

	kartesisch	zylindrisch	sphärisch
grad $\varphi$ ( $\nabla\varphi$ )	$= \frac{\partial\varphi}{\partial x}e_x$ $+ \frac{\partial\varphi}{\partial y}e_y$ $+ \frac{\partial\varphi}{\partial z}e_z$	$= \frac{\partial\varphi}{\partial r}e_r$ $+ \frac{1}{r} \frac{\partial\varphi}{\partial\alpha}e_\alpha$ $+ \frac{\partial\varphi}{\partial z}e_z$	$= \frac{\partial\varphi}{\partial r}e_r$ $+ \frac{1}{r} \frac{\partial\varphi}{\partial\vartheta}e_\vartheta$ $+ \frac{1}{r\sin\vartheta} \frac{\partial\varphi}{\partial\alpha}e_\alpha$
div $\mathbf{v}$ ( $\nabla \cdot \mathbf{v}$ )	$= \frac{\partial v_x}{\partial x}$ $+ \frac{\partial v_y}{\partial y}$ $+ \frac{\partial v_z}{\partial z}$	$= \frac{1}{r} \frac{\partial(r \cdot v_r)}{\partial r}$ $+ \frac{1}{r} \frac{\partial v_\alpha}{\partial\alpha}$ $+ \frac{\partial v_z}{\partial z}$	$= \frac{1}{r^2} \frac{\partial(r^2 \cdot v_r)}{\partial r}$ $+ \frac{1}{r\sin\vartheta} \frac{\partial(\sin\vartheta \cdot v_\vartheta)}{\partial\vartheta}$ $+ \frac{1}{r\sin\vartheta} \frac{\partial v_\alpha}{\partial\alpha}$
rot $\mathbf{v}$ ( $\nabla \times \mathbf{v}$ )	$= \begin{pmatrix} \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \\ \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \end{pmatrix} \begin{matrix} e_x \\ e_y \\ e_z \end{matrix}$	$= \begin{pmatrix} \frac{1}{r} \frac{\partial v_z}{\partial\alpha} - \frac{\partial v_\alpha}{\partial z} \\ \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \\ \frac{\partial(rv_\alpha)}{\partial r} - \frac{\partial v_r}{\partial\alpha} \end{pmatrix} \begin{matrix} e_r \\ e_\alpha \\ e_z \end{matrix}$	$= \frac{1}{r\sin\vartheta} \begin{pmatrix} \frac{\partial(v_\alpha \sin\vartheta)}{\partial\vartheta} - \frac{\partial v_\vartheta}{\partial\alpha} \\ \frac{1}{\sin\vartheta} \frac{\partial v_r}{\partial\alpha} - \frac{\partial(rv_\alpha)}{\partial r} \\ \frac{\partial(rv_\vartheta)}{\partial r} - \frac{\partial v_r}{\partial\vartheta} \end{pmatrix} \begin{matrix} e_r \\ e_\vartheta \\ e_\alpha \end{matrix}$
$\Delta\varphi$ ( $\nabla^2\varphi$ )	$= \frac{\partial^2\varphi}{\partial x^2}$ $+ \frac{\partial^2\varphi}{\partial y^2}$ $+ \frac{\partial^2\varphi}{\partial z^2}$	$= \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial\varphi}{\partial r} \right)$ $+ \frac{1}{r^2} \frac{\partial^2\varphi}{\partial\alpha^2}$ $+ \frac{\partial^2\varphi}{\partial z^2}$	$= \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial\varphi}{\partial r} \right)$ $+ \frac{1}{r^2 \sin\vartheta} \frac{\partial}{\partial\vartheta} \left( \sin\vartheta \frac{\partial\varphi}{\partial\vartheta} \right)$ $+ \frac{1}{r^2 \sin^2\vartheta} \frac{\partial^2\varphi}{\partial\alpha^2}$
$\Delta \mathbf{v}$	$= \Delta v_x e_x$ $+ \Delta v_y e_y$ $+ \Delta v_z e_z$	.....	.....

**Tabelle 4: Rechenregeln der Vektoranalysis und Integralsätze**

Zusammengesetzte Ausdrücke	Integralsätze
$\text{grad}(\varphi + \psi) = \text{grad} \varphi + \text{grad} \psi$ $\text{grad}(\varphi \cdot \psi) = \varphi \text{grad} \psi + \psi \text{grad} \varphi$ $\text{grad}(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{A} \cdot \nabla) \mathbf{B} + (\mathbf{B} \cdot \nabla) \mathbf{A} + \mathbf{A} \times \text{rot} \mathbf{B} + \mathbf{B} \times \text{rot} \mathbf{A}$ $\text{div}(\mathbf{A} + \mathbf{B}) = \text{div} \mathbf{A} + \text{div} \mathbf{B}$ $\text{div}(\varphi \cdot \mathbf{A}) = \varphi \cdot \text{div} \mathbf{A} + \mathbf{A} \cdot \text{grad} \varphi$ $\text{div}(\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot \text{rot} \mathbf{A} - \mathbf{A} \cdot \text{rot} \mathbf{B}$ $\text{rot}(\mathbf{A} + \mathbf{B}) = \text{rot} \mathbf{A} + \text{rot} \mathbf{B}$ $\text{rot}(\varphi \cdot \mathbf{A}) = \varphi \cdot \text{rot} \mathbf{A} + \text{grad} \varphi \times \mathbf{A}$ $\text{rot}(\mathbf{A} \times \mathbf{B}) = \mathbf{A} \cdot \text{div} \mathbf{B} - \mathbf{B} \cdot \text{div} \mathbf{A} + (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B}$ $\text{rot rot} \mathbf{A} = \text{grad div} \mathbf{A} - \nabla^2 \mathbf{A}$ $\text{div grad} \varphi = \nabla^2 \varphi = \Delta \varphi$ $\text{rot grad} \varphi = 0$ $\text{div rot} \mathbf{A} = 0$	<p><b>Satz von Gauß</b></p> $\int_V \text{div} \mathbf{B} \, dV = \oint_A \mathbf{B} \cdot d\mathbf{A}$ <p><b>Satz von Stokes</b></p> $\int_A \text{rot} \mathbf{B} \cdot d\mathbf{A} = \oint_C \mathbf{B} \cdot ds$ <p><b>Greensche Sätze</b></p> <p>1. Satz:</p> $\oint_A (\varphi \cdot \text{grad} \psi) \cdot d\mathbf{A} = \int_V \text{div}(\varphi \cdot \text{grad} \psi) \, dV$ $= \int_V \varphi \nabla^2 \psi \, dV + \int_V \text{grad} \varphi \cdot \text{grad} \psi \, dV$ <p>2. Satz:</p> $\int_V (\varphi \nabla^2 \psi - \psi \nabla^2 \varphi) \, dV =$ $= \oint_A (\varphi \text{grad} \psi - \psi \text{grad} \varphi) \cdot d\mathbf{A}$
$\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am}$ $\epsilon_0 = 8.86 \cdot 10^{-12} \text{ As/Vm}$	