Grundlagen und Begriffsbestimmungen

$A := \{ X; \blacktriangledown; \blacktriangle; \sim; n\blacktriangledown \}$ → Algebra

$X := \{0; 1\}$ → Schaltalgebra

$S := \{ \{0;1\}, \vee, \wedge, \neg, 0, 1 \}$ → Symbolalgebra

2-wertige Operatoren:

Disjunktion: $\vee$, $+$

Konjunktion: $\wedge$, $\cdot$

Negation: $\neg$, $-$

Kommutativ

$$x_1 + x_0 = x_0 + x_1$$

$$x_1 x_0 = x_0 x_1$$

Assoziativ

$$(x_2 + x_1) + x_0 = x_2 + (x_1 + x_0)$$

$$(x_2 \cdot x_1) \cdot x_0 = x_2 \cdot (x_1 \cdot x_0)$$

Distributiv

$$x_2 + (x_1 \cdot x_0) = (x_2 + x_1) \cdot (x_2 + x_0)$$

$$x_2 \cdot (x_1 + x_0) = (x_2 \cdot x_1) + (x_2 \cdot x_0)$$

Adjunktiv / Absorptiv

$$x_1 \cdot (x_1 + x_0) = x_1$$

$$x_1 + (x_1 \cdot x_0) = x_1$$

neutrale Elemente

$x + 0 = x \qquad x \cdot 0 = 0$

$x + 1 = 1 \qquad x \cdot 1 = x$

Idempotenz

$x + x = x$

$x \cdot x = x$

Negation

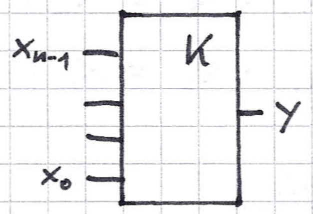
$\bar{\bar{x}} = x$

$x + \bar{x} = 1$

$x \cdot \bar{x} = 0$

Bool'sche Funktion

$$y = f(x_{n-1}, \dots, x_\varkappa, \dots, x_0)$$



$x, y \in X$

$\underline{x} = (x_{k-1}, \dots, x_0)$ Eingangsbelegung $\rightarrow \ell = 2^k$

Vektor

$$\varepsilon = \sum_{x=0}^{k} x_\varkappa \, 2^k$$

ε	2^2 x_2	2^1 x_1	2^2 x_0	y
0	0	0	0	1
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$$\underline{x} = \begin{bmatrix} x_0 \\ \vdots \\ x_\varepsilon \\ \vdots \\ x_{e-1} \end{bmatrix} = \begin{bmatrix} x_{0,n-1} & & \\ & \ddots & \\ & & x_{e-1,0} \end{bmatrix}$$

$$[y_0, \dots, y_\alpha, \dots, y_{a-1}] \qquad \underline{y}^f = \begin{bmatrix} y_0 \\ \vdots \\ y_\varepsilon \\ \vdots \\ y_{e-1} \end{bmatrix}$$

Ausgangsbelegung aller e Eingangsbelegung

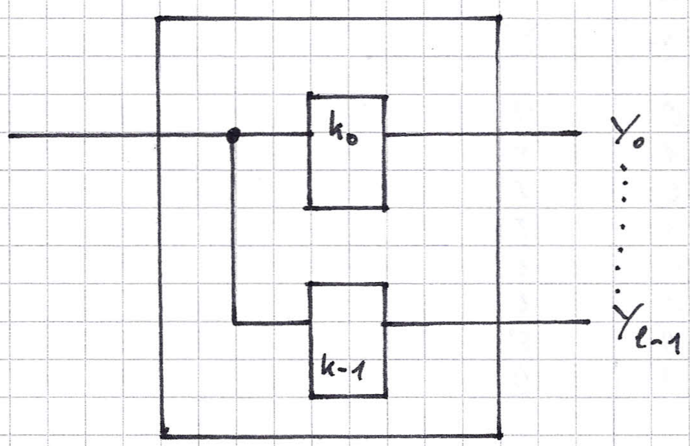
<u>Ausgangsalphabet:</u> $a = 2^e = 2^{2^k}$

$$\underline{y} = [y_0, \dots, y_{a-1}] = \begin{bmatrix} y_{0,0} & \dots & y_{a-1,0} \\ \vdots & & \vdots \\ y_{0,e-1} & \dots & y_{a-1,e-1} \end{bmatrix}$$

<u>Augangsbelegung:</u>

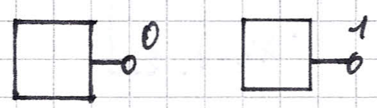
$$\underline{y}^f = \begin{bmatrix} \underline{y}_0 \\ \vdots \\ \underline{y}_\varepsilon \\ \vdots \\ y_{e-1} \end{bmatrix} = \begin{bmatrix} y_{0,0} & \dots & y_{0,e-1} \\ \vdots & & \vdots \\ y_{e-1,0} & \dots & y_{e-1,e-1} \end{bmatrix}$$

$\hookrightarrow$ a^e mögliche Funktionen



Funktionen für $k=0$

$a = 2^{2^0} = 2$



Funktionen für $k=1$

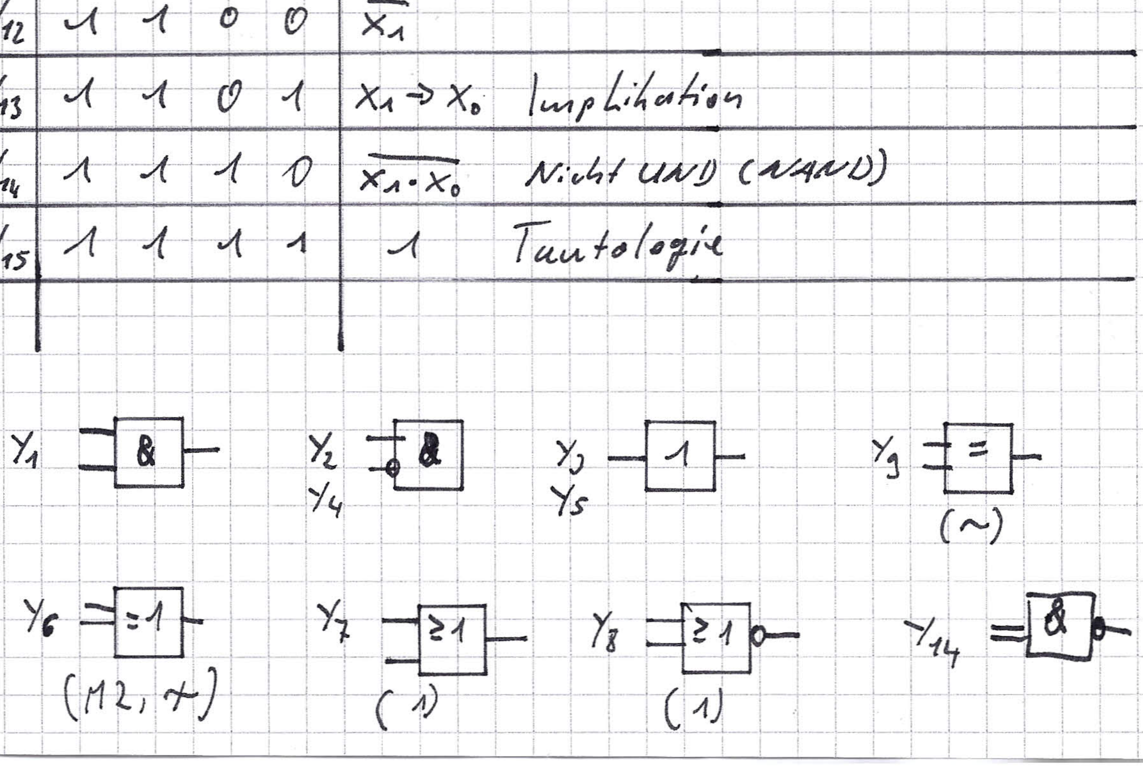
$a = 2^{2^1} = 4$

ε	0	1		
x_0	0	1	Funktion	Name
Y_0	0	0	0	Kontradiktion
Y_1	0	1	x_0	Identität
Y_3	1	0	$\overline{x}_0 = \neg x_0$	Negation
Y_4	1	1	1	Tautologie

2 Eingänge

$n = 2 \qquad a = 2^{2^2} = 16$

ε	0	1	2	3		
x_1	0	0	1	1		
x_0	0	1	0	1	y	
y_0	0	0	0	0	0	Kontradiktion
y_1	0	0	0	1	$x_1 \cdot x_0$	Konjunktion, UND (AND)
y_2	0	0	1	0	$x_1 \nrightarrow$	Inhibition
y_3	0	0	1	1	x_1	Identität
y_4	0	1	0	0	$x_1 \nleftarrow x_0$	Inhibition
y_5	0	1	0	1	x_0	Identität
y_6	0	1	1	0	$x_1 \not\sim x_0$	Antivalenz
y_7	0	1	1	1	$x_1 \bullet x_0$	ODER (OR)
y_8	1	0	0	0	$\overline{x_1 \bullet x_0}$	Nicht ODER (NOR)
y_9	1	0	0	1	$x_1 \sim x_0$	Äquivalenz
y_{10}	1	0	1	0	$\overline{x_0}$	
y_{11}	1	0	1	1	$x_0 \rightarrow x_1$	Implikation
y_{12}	1	1	0	0	$\overline{x_1}$	
y_{13}	1	1	0	1	$x_1 \rightarrow x_0$	Implikation
y_{14}	1	1	1	0	$\overline{x_1 \cdot x_0}$	Nicht UND (NAND)
y_{15}	1	1	1	1	1	Tautologie



Funktionen mit 3 Eingängen

$a = 2^{2^3} = 256$ $\qquad \varepsilon \rightarrow$ Index der Eingangsbelegung

ε	x_2	x_1	x_0	OR		Ex-OR 1 u.n. 1		2 u.n. 2		Antivalenz		Aquivalenz	
0	0	0	0	0	0	0	0	0	0	0	0		
1	0	0	1	1	1	1	1	0	0	1	1		
2	0	1	0	1	1	1	1	0	0	1	1		
3	0	1	1	1	1	0	0	1	1	0	0		
4	1	0	0	1		1		0		1			
5	1	0	1	1		0		1		0			
6	1	1	0	1		0		1		0			
7	1	1	1	1		0		0		1			
Symbol				≥ 1		$\boxed{\not\sim}$ $=1$		$\boxed{\&}$ $=2$		$\boxed{=1}$ $\not\sim$ M2		$= \sim$	

Def.: $Y = \underset{\kappa=0}{\overset{k-1}{\not\sim}} x_\kappa = x_{k-1} \not\sim x_{k-2} \not\sim \dots \not\sim x_0$

$Y = \underset{\kappa=0}{\overset{k-1}{\sim}} x_\kappa = x_{k-1} \sim x_{k-2} \sim \dots \sim x_0$

Antivalenz	Äquivalenz
$x \not\sim 0 = x$	$x \sim 0 = \overline{x}$
$x \not\sim 1 = \overline{x}$	$x \sim 1 = x$
$x \not\sim x = 0$	$x \sim x = 1$

$x_1 \not\sim x_0 = \overline{x_1 \sim x_0}$

$x_1 \not\sim \overline{x_0} = \overline{x_1} \not\sim x_0 = x_1 \sim x_0$

Eingangs- und Ausgangsseitig

$x, y \in \{0, 1\}$ $\quad$ d = don't care

$x \in \{0, 1, -\}$ $\quad$ $y \in \{0, 1, d\}$

x_1	x_0
-	1

x_1	x_0
0	1

x_1	x_0
1	1

NOR

ε	x_1	x_0	y_0	y_0'
0	0	0	1	1
1	0	1	0	d
2	1	0	0	0
3	1	1	0	0

$d = 0$

$y_0' = \bar{x}_1 \bar{x}_0 = y_0$

$d = 1$

$y_0' = \bar{x}_1 \bar{x}_0 + \bar{x}_1 x_0$

$- \bar{x}_1 \not\equiv y_0$

$$y = \bar{x}_1 x_0 + x_1 x_0 = (\bar{x}_1 + x_1) x_0 = x_0$$

Wichtige Gesetze der Schaltalgebra

De Morganische Theorem / Inversionssatz

2 Variablen: $\overline{x_1 x_0} = \bar{x}_1 + \bar{x}_0$

$\overline{x_1 + x_0} = \bar{x}_1 \cdot \bar{x}_0$

k Variablen:

$$\overline{\prod_{\varkappa=0}^{k-1} x_\varkappa} = \sum_{\varkappa=0}^{k-1} \overline{x_\varkappa}$$

$$\overline{\sum_{\varkappa=0}^{k-1} x_\varkappa} = \prod_{\varkappa=0}^{k-1} \overline{x_\varkappa}$$

allgemein:

$$\overline{f(x_\varkappa, \overline{x_\varkappa}, \cdot, +, 1, 0)} = f(\overline{x_\varkappa}, x_\varkappa, +, \cdot, 0, 1)$$

$$y = \overline{\overline{x_6+x_5} + \overline{\overline{x_3\,\overline{\overline{(x_4+\overline{x_2})}}} \cdot \overline{\overline{x_1}+x_0}}}$$

$$= (x_6+x_5) \cdot \overline{(x_3\,\overline{\overline{(x_4+\overline{x_2})}})} \cdot (\overline{\overline{x_1}+x_0})$$

$$= (x_6+x_5)\,(\overline{x_3} + (x_4+\overline{x_2})) \cdot (x_1 \cdot \overline{x_0})$$

$$= (x_6+x_5)\,(\overline{x_3} + x_4 + \overline{x_2})\, x_1 \overline{x_0}$$

$$y = \overline{\overline{(x_6+x_5)} + \overline{\left[\overline{(x_3\,\overline{\overline{(x_4+\overline{x_2})}})}\;\overline{(\overline{x_1}+x_0)}\right]}}$$

$$= (x_6+x_5) \cdot \left[(\overline{x_3} + (x_4+\overline{x_2})) \cdot (x_1 \cdot \overline{x_0})\right]$$

$$= (x_6+x_5)\,(\overline{x_3} + x_4 + \overline{x_2})\, x_1 \overline{x_0}$$

Shannon-Theorem, Entwicklungssatz

$$y = f(x_{k-1}, \dots, x_\varkappa, \overline{x}_\varkappa, \dots, x_0)$$
$$= x_\varkappa \cdot f(x_{k-1}, \dots, 1, 0, \dots x_0) +$$
$$\overline{x}_\varkappa \cdot f(x_{k-1}, \dots, 0, 1, \dots x_0)$$

$$y = x_2 x_1 + \overline{x_1} x_0$$
$$= x_2\,(1 \cdot x_1 + \overline{x}_1 x_0) + \overline{x}_2\,(0 \cdot x_1 + \overline{x}_1 x_0)$$

Dualitätsprinzip / Shannon'sche Gesetze

duale Aussage

$0 \longleftrightarrow 1$

$\cdot \longleftrightarrow +$

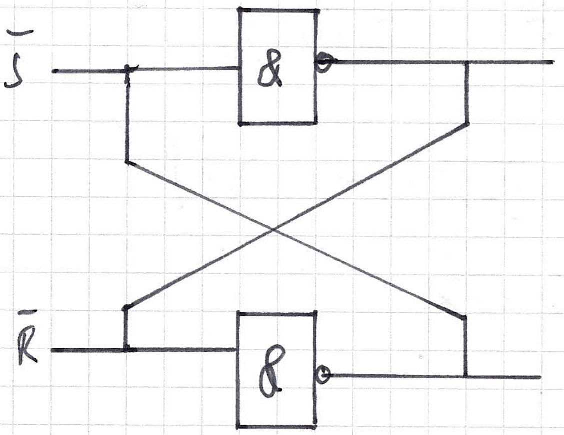
x_1	x_0	$x_1 \cdot x_0$
0	0	0
0	1	0
1	0	0
1	1	1

$\Longleftrightarrow$

x_1	x_0	$x_1 + x_0$
1	1	1
1	0	1
0	1	1
0	0	0

NAND - Basissystem

RS - FF



ε	S	R	Q	$\bar{Q}$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	d
7	1	1	1	d

DNF → Disjunktive Verknüpfung → ODER - Verknüpfung

$Y = f(x) = \bar{x}_1 \bar{x}_0 + \bar{x}_2 x_1 + x_2 x_1 x_0$

KNF → Konjunktive Verknüpfung → UND - Verknüpfung

$Y = f(x) = (x_1 + \bar{x}_0)(\bar{x}_2 + \bar{x}_1 + x_0)$

DNF

KNF

$$y = f(x) = \overline{x_1}\,\overline{x_0} + \overline{x_2}\,x_1 + x_2 x_1 x_0 = (x_1 + \overline{x_0})(\overline{x_2} + \overline{x_1} + x_0)$$

KDNF

Minterme m_E

$m_7 = x_2 x_1 x_0$

minimierte DNF

minimale DNF

PDK

kanonische

in jedem Term alle Eingangsvariablen

Minimierung

KKNF

Maxterme M_E

$M_E = \overline{x_2} + \overline{x_1} + x_0$

minimierte KNF

minimale KNF

PKK

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Zusammenhänge zwischen ~~Min~~ KDNF und KKNF

1.) $\overline{m_i} = M_i$, $m_i = \overline{M_i}$ $m_1: \bar{x}_2 \bar{x}_1 x_0$

$\downarrow$

morgan

$\downarrow$

$:= \overline{\bar{x}_2 \bar{x}_1 x_0} = x_2 + x_1 + \bar{x}_0$

$y_0 m_0 + y_1 m_1 \dots$

$m_0 + m_1 + m_2$

$1 \cdot (\bar{x}_2 \bar{x}_1 \bar{x}_0)$

NAND / NOR

$f(x)$

$\xrightarrow{1}$ (K) DNF $\quad\quad$ $\xrightarrow{0}$ (K) KNF

(K) DNF → $\overline{\overline{f(x)}}$ (2x negirane) → D-NAND-NF

(K) DNF → $\overline{\overline{\overline{\overline{f(x)}}}}$ (4x negirane) → D-NOR-NF

(K) KNF → $\overline{\overline{f(x)}}$ → K-NOR-NF

(K) KNF → $\overline{\overline{\overline{\overline{f(x)}}}}$ → K-NAND-NF

DNF $\quad f(x) = x_2 x_1 + \bar{x}_2 x_0$

NAND: $\overline{\overline{x_2 x_1 + \bar{x}_2 x_0}} = \overline{\overline{x_2 \cdot x_1}\ \overline{\bar{x}_2 x_0}}$

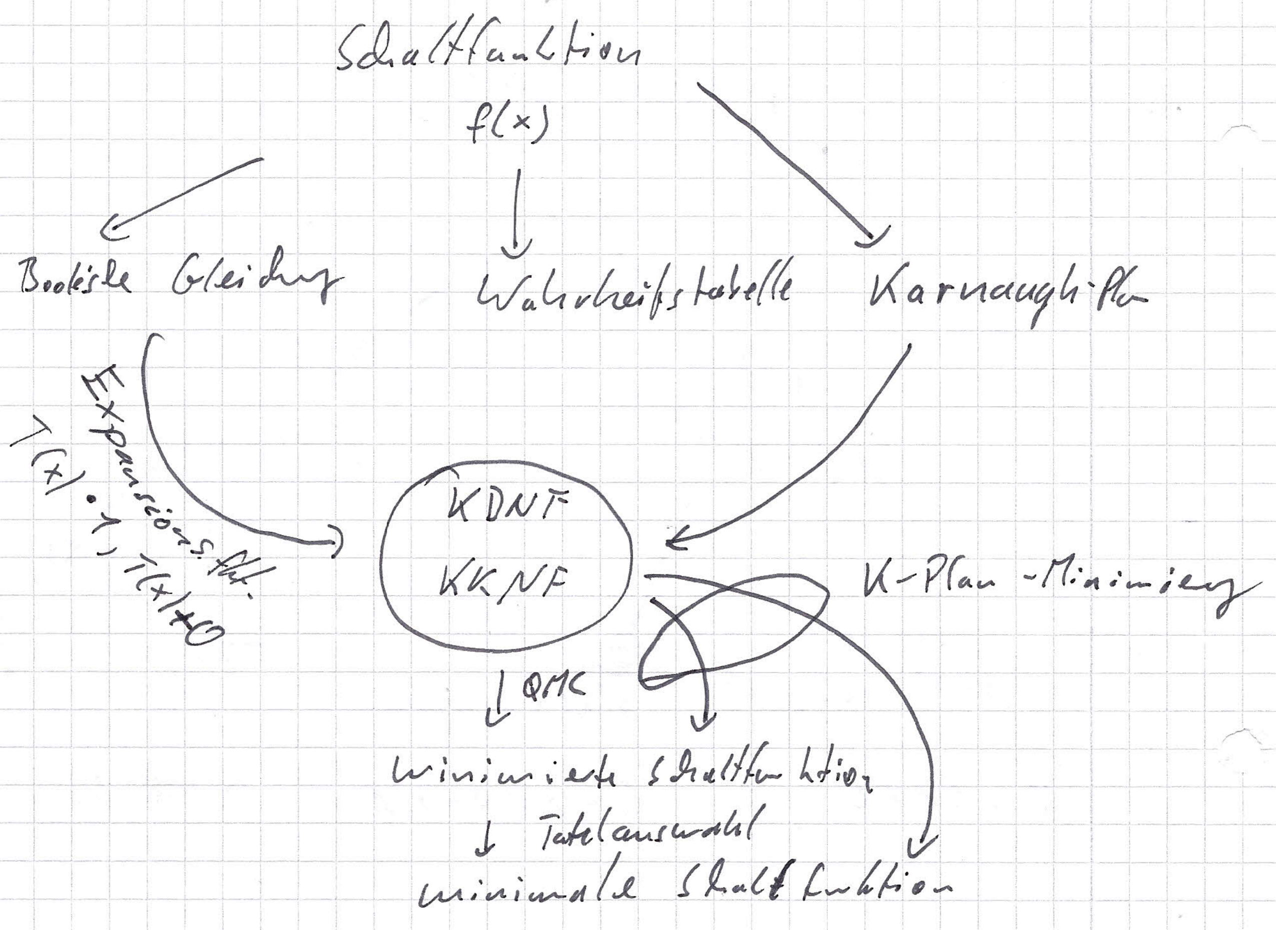
NOR: $\overline{\overline{\overline{\overline{x_2 x_1 + \bar{x}_2 x_0}}}} = \overline{\overline{\overline{\bar{x}_2 + \bar{x}_1} + \overline{x_2 + \bar{x}_0}}}$

KNF: $f(x) = (\bar{x}_2 + \bar{x}_1)(x_2 + x_0)$

NOR: $\overline{\overline{(\bar{x}_2 + \bar{x}_1)(x_2 + x_0)}} = \overline{\overline{\bar{x}_2 + \bar{x}_1} + \overline{x_2 + x_0}}$

NAND: $\overline{\overline{\overline{\overline{(\bar{x}_2 + \bar{x}_1)(x_2 + x_0)}}}}$

$\overline{\overline{\overline{x_2 x_1} \cdot \overline{\bar{x}_2 \bar{x}_0}}}$



QMC → Quine / Mc Cluskey

⇒ Gray-Code : Hamming-Distanz = 1

ε	x_3	x_2	x_1	x_0
	0	0	0	0
	0	0	0	1
	0	0	1	0
	0	0	1	1
	0	1	0	0
	0	1	0	1
	0	1	1	0

x_1

x_0

	Y_C		0	
x_2	1	1		

DNF

Blockbildung mit

$Y_C = 1$

don't care $d = 1$

Blockgröße $B = 2^\sigma$

KNF

Blockbildung mit $Y_E = 0$

don't care $d = 0$

$\wedge$ $\vee$

$\cdot$ $+$

$x_2 \cdot \bar{x}_1$

x_2

x_0

	0_0	1_1	1_5	0_4
x_1	1_2	0_3	0_7	1_6

$Y = \bar{x}_1 x_0 + x_1 \bar{x}_0$

x_2

x_0

		0	d	0	0
x_1		0	1	1	d
x_1	x_3	d	1	1	0
	x_3	0	0	d	0

$Y = x_0 x_1$

x_2

x_0

0	1	0	0
0	d	d	1
1	d	d	0
0	0	1	0

x_1, x_3

$$Y = \bar{x}_3 \bar{x}_2 x_0 + \bar{x}_3 x_2 x_1 + x_3 x_2 x_0 + x_3 \bar{x}_2 x_1$$

x_0

1	1	1	1
1	1	0	0
1	1	1	1
0	0	1	1

x_1, x_3, x_2

$$\underline{Y = \bar{x}_3 \bar{x}_1 + \bar{x}_2 x_1 + x_3 x_2}$$

$\Downarrow$ einfachen

x_2

x_0 x_0

1	0	0	0	0	1	0	1
0	1	0	1	1	1	1	0
0	1	1	1	1	0	1	0
1	0	1	0	0	0	0	1

x_1, x_3, x_4

$$\bar{x}_2 \bar{x}_1 \bar{x}_0 + \bar{x}_2 x_1 x_0 + \bar{x}_4 x_3 x_2 x_0 + x_2 x_1 \bar{x}_0 + x_4 \bar{x}_3 x_2 x_0$$

Auslesen der minimalen KNF

		x_0	x_0, x_2	x_2
	0	1	1	0
x_1	1	0	0	1

$(x_1 + x_0)(\overline{x_1} + \overline{x_0})$

		x_0	x_0, x_2	x_2
	0	1	1	0
x_1	0	0	1	1

$(x_1 + x_0)(x_2 + \overline{x_1})$

		x_0	x_0, x_2	x_2
	1	0	d	d
x_1	1	0	d	d

$Y = \overline{x_0}$

Quine und Mc Clusky QMC

Stufenweise Blockbildung

DNF: $x_1 x_0 + x_1 \overline{x_0} = x_1 (x_0 + \overline{x_0}) = x_1 \cdot 1$

KNF: $(x_1 + x_0)(x_1 + \overline{x_0}) = x_1 (x_0 \overline{x_0}) = x_1 + 0$

<u>Ergebnis:</u>

minimierte DNF / KNF:

Ausgangspunkt KDNF

Beispiel: $k = 3$ $\quad f(x) = \sum 0, 2, 4, 5, 6, 7$

		x_0	x_0, x_2	x_2
	$_0$ 1	$_1$ 0	$_5$ 1	$_4$ 1
x_1	$_2$ 1	$_3$ 0	$_7$ 1	$_6$ 1

$y = x_2 + \overline{x_0}$

$f(x) = \overline{x_2}\,\overline{x_1}\,\overline{x_0} + \overline{x_2}\, x_1 \overline{x_0} + x_2 \overline{x_1}\,\overline{x_0} + x_2 \overline{x_1} x_0 + x_2 x_1 \overline{x_0} + x_2 x_1 x_0$

modifizierte Minterme

$f(x) = (000) + (010) + (100) + (101) + (110) + (111)$

ε	x_2	x_1	x_0	
0	0	0	0	✓
2	0	1	0	✓
4	1	0	0	✓
5	1	0	1	✓
6	1	1	0	✓
7	1	1	1	

ε	x_2	x_1	x_0
0,2	0	–	0
0,4	–	0	0
2,6	–	1	0
4,5	1	0	–
4,6	1	–	0
5,7	1	–	1
6,7	1	1	–

ε	x_2	x_1	x_0
0,2,4,6	–	–	0
4,5,6,7	1	–	–

$$Y = \bar{x}_0 + x_2$$

Ausgangsfunktion: KKNF Beispiel: $f(x) = \prod 10, 11, 12, 13, 14, 15$

		x_1	x_1		
	1_0	1_2	0_3	1_4	
x_0	1_1	1_3	0_{11}	1_{10}	
x_0	1_6	1_7	0_{15}	0_{14}	x_2
	1_4	1_5	0_{13}	0_{12}	x_2
			x_3	x_3	

$$Y = (\bar{x}_3 + \bar{x}_1)(\bar{x}_3 + \bar{x}_2)$$

ε	x: 3	2	1	0
10	1	0	1	0
12	1	1	0	0
11	1	0	1	1
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

ε	x: 3	2	1	0
10,11	1	0	1	–
10,14	1	–	1	0
12,13	1	1	0	–
12,14	1	1	–	0
11,15	1	–	1	1
13,15	1	1	–	1
14,15	1	1	1	–

ε	x: 3	2	1	0
10,11,14,15	1	–	1	–
12,13,14,15	1	1	–	–

$x \to 0$

$\bar{x} \to 1$

$$(\bar{x}_3 + \bar{x}_1)(\bar{x}_3 + \bar{x}_2)$$

1.) minimale DNF $\quad Y = f(x) = \sum 0, 2, 3, 5, 7$

	$x_2\,x_1\,x_0$	
0	0 0 0	✓
2	0 1 0	✓
3	0 1 1	✓
5	1 0 1	
7	1 1 1	✓

	$x_2\,x_1\,x_0$
0,2	0 – 0
2,3	0 1 –
3,7	– 1 1
5,7	1 – 1

Primimplikanten → $\bar{x}_2\bar{x}_0 + \bar{x}_2 x_1 + x_1 x_0 + x_2 x_0$

	$\bar{x}_2\bar{x}_0$	$\bar{x}_2 x_1$	$x_1 x_0$	$x_2 x_0$
$\bar{x}_2\bar{x}_1\bar{x}_0$	X			
$\bar{x}_2 x_1 \bar{x}_0$	X	X		
$\bar{x}_2 x_1 x_0$		X	X	
$x_2 \bar{x}_1 x_0$				X
$x_2 x_1 x_0$			X	X
	P_{DK1}	P_{D1}	P_{D2}	P_{DK1}

Kernprimimplikant

~~Prim~~ P_{DK1}

Primimplikant

P_D

minimale DNF: $Y = P_{DK1} + P_{DK2}$ → P_{D1} / → P_{D2}

$Y = \bar{x}_2\bar{x}_0 + x_2 x_0 +$ → $\bar{x}_2 x_1$ / → $x_1 x_0$

Kurzform:

$Y = \binom{0}{2} + \binom{5}{7} +$ → $\binom{2}{3}$ / → $\binom{3}{7}$

2. Minimale KNF: $k=4$ $\quad Y = f(x) = \prod 5, 10, 12, 13, 14$

ε	x_3	x_2	x_1	x_0
5	0	1	0	1
10	1	0	1	0
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0

	x_3	x_2	x_1	x_0
5, 13	-	1	0	1
10, 14	1	-	1	0
12, 13	1	1	0	-
12, 14	1	1	-	0

$$(\overline{x_2} + x_1 + \overline{x_0})(\overline{x_3} + \overline{x_1} + x_0)(\overline{x_3} + \overline{x_2} + x_1)$$

$$(\overline{x_3}\ \overline{x_2}\ x_0)$$

	5, 13	10, 14	12, 13	12, 14
5	X			
10		X		
12			X	X
13	X		X	
14		X		X
	PKK1	PKK2	PK1	PK2

~~$\prod 5, 10$~~

$$Y = \binom{5}{13}\binom{10}{14} \to \binom{12}{13}, \binom{12}{14}$$

$$Y = (\overline{x_2} + x_1 + \overline{x_0})(\overline{x_3} + \overline{x_1} + x_0) \to (\overline{x_3} + \overline{x_2} + x_1), (\overline{x_3} + \overline{x_2} + x_0)$$

Quine McCluskey mit Don't care:

$$f(x) = \sum_{1}^{\sim 1} m_\varepsilon y_\varepsilon \;:\; y_\varepsilon \begin{cases} 1 \text{ für } \varepsilon \in E = \{2, 5, 9, 14\} \\ d \text{ für } \varepsilon \in D = \{3, 7, 11, 15\} \end{cases}$$

x_0

	0	0	1	0
	1	d	d	0
x_1	0	d	d	1
	0	1	0	0	x_3

x_2

Vorgehensweise

- bei QMC don't care in Blockbildung einbeziehen.
- TAV ohne don't cares

ε	x_3	x_2	x_1	x_0	
2	0	0	1	0	✓
3	0	0	1	1	✓
5	0	1	0	1	✓
9	1	0	0	1	✓
7	0	1	1	1	✓
11	1	0	1	1	✓
14	1	1	1	0	✓
15	1	1	1	1	✓

	x_3	x_2	x_1	x_0	
2,3	0	0	1	–	
3,7	0	–	1	1	
3,11	–	0	1	1	✓
5,7	0	1	–	1	
9,11	1	0	–	1	
7,15	–	1	1	1	✓
11,15	1	–	1	1	✓
14,15	1	1	1	–	

	x_3	x_2	x_1	x_0
3,11 7,15	–	–	1	1

m_i	2, 3	5, 7	14, 15	3, 7, 11, 15	9, 11
2	x				
5		x			
9					x
14			x		

minimale DNF:

$$Y = \begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}5\\7\end{pmatrix} + \begin{pmatrix}9\\11\end{pmatrix} + \begin{pmatrix}14\\15\end{pmatrix}$$

$$= \overline{x}_3 \overline{x}_2 x_1 + \overline{x}_3 x_2 x_0 + x_3 \overline{x}_2 x_0 + x_3 x_2 x_1$$

5. Mehrstufige Realisierungen – Faktorisierung

5.1. Einleitung

$$y = f(x) = \sum y_c m_c = \prod (y_c + M_c)$$

$n = 3 \qquad y = \sum 4,6,7 = \prod 0,1,2,3,5$

		x_0	x_0	
	0	0	0	1
x_1	0	0	1	1
			x_2	x_2

DNF

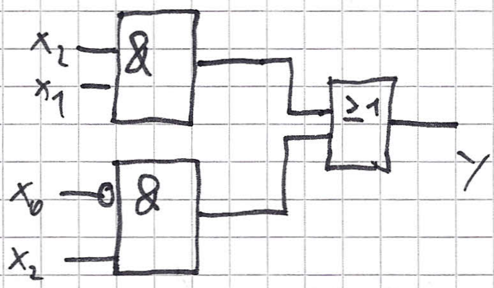
$y = x_2 x_1 + x_2 \overline{x_0}$

KNF

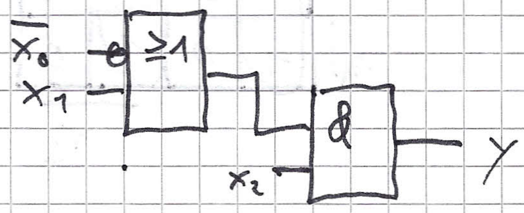
$y = x_2 \cdot (\overline{x_0} + x_1)$

• **Krause Logik**

$y = x_2 x_1 + x_2 \overline{x_0}$

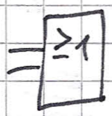


$y = x_2 \cdot (\overline{x_0} + x_1)$



• **Realisierung in ein Basissystem**

Nand 

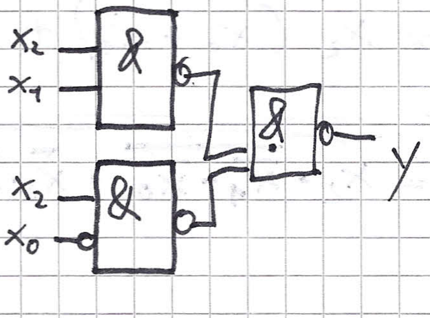
NOR 

minimale DNF:

$y = x_2 x_1 + x_2 \overline{x_0}$

$= \overline{\overline{x_2 x_1 + x_2 \overline{x_0}}}$

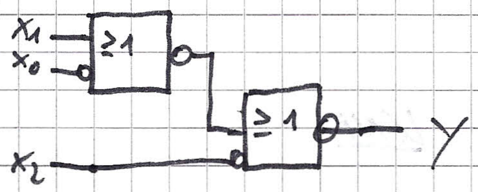
$= \overline{\overline{x_2 x_1} \cdot \overline{x_2 \overline{x_0}}}$



minimale KNF:

$$Y_2 = x_2 \cdot (x_1 + \bar{x}_0)$$

$$Y_2 = \overline{\bar{x}_2 + \overline{(x_1 + \bar{x}_0)}}$$



5.2. Zergliedernde Faktorisierung

	x_0				
	1	0	1	1	
x_1	1	0	1	0	
			x_2		

$$Y = x_2 x_0 + \bar{x}_2 \bar{x}_0 + x_2 \bar{x}_1 \quad | \text{ ausklammern}$$

$$Y = x_2 (x_0 + \bar{x}_1) + \bar{x}_2 \bar{x}_0 \quad | \text{ negieren}$$

$$Y = \overline{\overline{x_2 (x_0 + \bar{x}_1) + \bar{x}_2 \bar{x}_0}}$$

$$Y = \overline{\overline{x_2 \overline{\overline{(x_0 + \bar{x}_1)}} + \bar{x}_2 \bar{x}_0}}$$

$$Y = \overline{\overline{x_2 \cdot \overline{x_1 \cdot \bar{x}_0} \cdot \bar{x}_2 \cdot \bar{x}_0}}$$

Multiplexer:

MUX: $Y = \overbrace{\overline{A}_{k-1} \cdots \overline{A}_0 \cdot D_0}^{m_0 Y_0} + \overbrace{A_{k-1} \cdot \overline{A}_0 \cdot D_1}^{m_1 Y_1} + \dots$

KDNF: $Y = \sum m_\varepsilon \cdot Y_\varepsilon = \overbrace{\overline{X}_{k-1} \cdots \overline{X}_0}^{m_0} \cdot Y_0 + \overbrace{\overline{X}_{k-1} \cdots X_0}^{m_1} \cdot Y_1 + \dots$

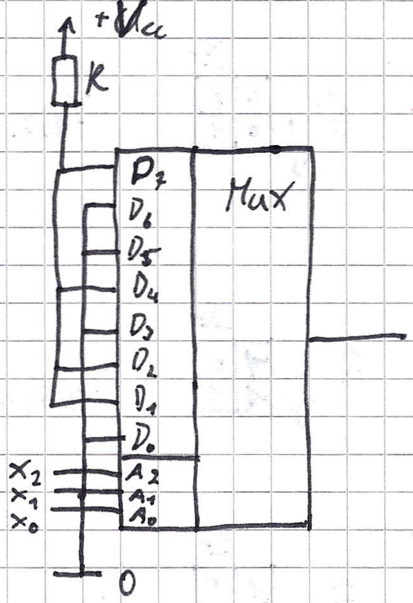
Realisierung von Schaltfunktionen mit MUX

- $X_\mathcal{H}$ adressiert Funktionswert $\to X_\mathcal{H} \to A_\mathcal{H}$
  ↳ Anzahl Adresseingänge = Anzahl Variable
- X_ε adressiert Eingangsleitung $Y_\varepsilon = \{0,1\}$ an D_ε

Beispiel:

$k = 3 \quad Y = \sum 1,2,4,7$

		X_0	X_0	
	0_0	1_1	0_5	1_4
X_1	1_2	0_3	1_7	0_6
			X_2	X_2



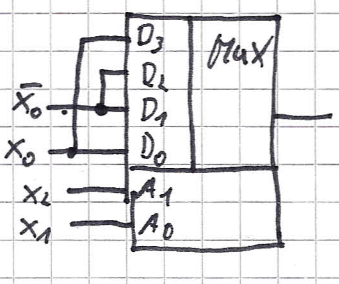
Anzahl Adr-Eing < Anzahl Variable

$k = 3 \quad Y = \sum 1,2,4,7$

		X_0	X_2	X_2
	0	1 (0,1 → 0)	0	1 (4,5 → 2)
X_1	1	0 (2,3 → 1)	1	0 (6,7 → 3)

±1

±2



$\overline{X}_2\overline{X}_1$	X_0
0	1

$Y_{0,1} = X_0$

$\overline{X}_2 X_1$	X_0
1	0

$Y_{2,3} = \overline{X}_0$

$X_2\overline{X}_1$	X_0
1	0

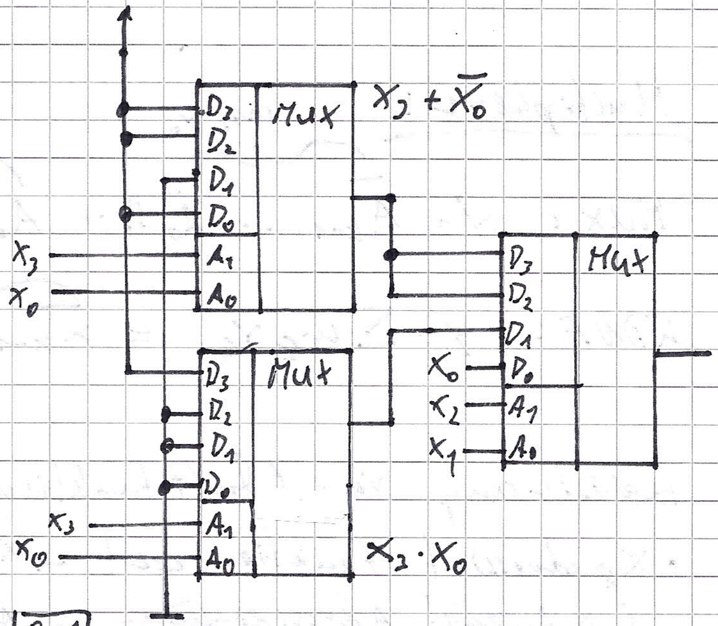
$Y_{4,5} = \overline{X}_0$

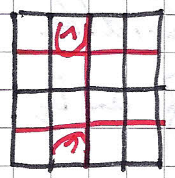
$X_2 X_1$	X_0
0	1

$Y_{6,7} = X_0$

	x_0		x_2
0	1	0	1
0	0	0	1
0	1	1	1
0	1	1	1

x_1, x_3



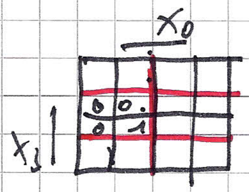


$\hat{=}$

0	1
0	1

x_3, $\overline{x_0}$

$Y_{0,1,8,9} = x_0$

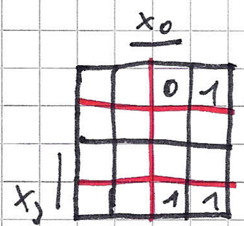


$\hat{=}$

	x_0
0	0
0	1

x_3

$Y_{2,3,10,11} = x_3 \cdot x_0$



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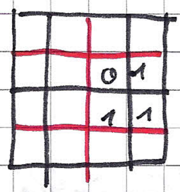
	x_0
1	0
1	1

x_3

$Y_{4,5,12,13} = x_3 + \overline{x_0}$



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	x_0
1	0
1	1

x_3

$Y_{6,7,14,15} = x_3 + \overline{x_0}$