Lösung erfolgt in Zylinderkoordinaten

$x = r\cos\varphi \qquad y = r\sin\varphi \qquad z = z$

$z = x^2 + y^2 \;\hat{=}\; z = r^2$

$z = x^2 + y^2 + 4 \;\hat{=}\; z = r^2 + 4$

$$m = \int_{\varphi=0}^{2\pi} \left(\int_{z=0}^{4} \left(\int_{r=0}^{\sqrt{z}} r\,dr \right) dz \right) d\varphi + \int_{\varphi=0}^{2\pi} \left(\int_{z=4}^{10} \left(\int_{r=\sqrt{z-4}}^{\sqrt{z}} r\,dr \right) dz \right) d\varphi$$

$$= 2\pi \left(\int_{r=0}^{4} \left(\frac{1}{2} \left[r^2 \right]_0^{\sqrt{z}} \right) dz \right) + 2\pi \left(\int_{z=4}^{10} \left(\frac{1}{2} \left[r^2 \right]_{\sqrt{z-4}}^{\sqrt{z}} \right) dz \right)$$

$$= \pi \left(\int_{r=0}^{4} z\,dz \right) + \pi \left(\int_{z=4}^{10} 4\,dz \right)$$

$$= \pi \left[\frac{1}{2} z^2 \right]_0^4 + 4\pi \left[z \right]_4^{10}$$

$$= 8\pi + 24\pi$$

$$= 32\pi$$