5.) $T(x;y;z) = xy + 2yz \qquad (x;y;z) \in B \subseteq \mathbb{R}^3$

$P_0\,(2;1;-1) \qquad \vec{e_s} = (1;1;1)^T$

$T_z = 2y$

$T_y = x + 2z$

$T_z = y$

$$\frac{\partial z}{\partial \vec{a}} = \frac{1}{\sqrt{a_1^2 + a_2^2 + a_3^2}} \cdot (a_1 T_x + a_2 T_y + a_3 T_z)$$

$$= \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} \cdot \{1 \cdot 1 + 1 \cdot [2 + 2 \cdot (-1)] + 1 \cdot 2 \cdot 1\}$$

$$= \underline{\underline{\sqrt{3}}}$$

6.)

a) $z = 3x^2 y + 4y^3 - 3x^2 - 12y^2 + 1$

$z_x = 6xy - 6x$

$z_y = 3x^2 + 12y^2 - 24y$

$z_{xx} = 6y - 6$

$z_{yy} = 24y - 24$

$z_{xy} = 6x = z_{yx}$

$0 \stackrel{!}{=} 6xy - 6x = 6x(y-1) \rightsquigarrow x_1 = 0$

$\rightsquigarrow y_1 = 1$

$0 \stackrel{!}{=} 3x^2 + 12y^2 - 24y$