

$$b) z = \sqrt{xy} \quad P_0(2, 2, 2) \quad \alpha = \frac{2}{3}\pi$$

$$z_x = \frac{1}{2}(xy)^{-\frac{1}{2}} \cdot y$$

$$z_y = \frac{1}{2}(xy)^{-\frac{1}{2}} \cdot x$$

$$\frac{\partial z}{\partial \alpha} = z_x \cdot \cos \alpha + z_y \cdot \sin \alpha$$

$$= \frac{y \cdot \cos \alpha + x \cdot \sin \alpha}{2\sqrt{xy}}$$

$$= \frac{2 \cdot \cos\left(\frac{2}{3}\pi\right) + 2 \cdot \sin\left(\frac{2}{3}\pi\right)}{2 \cdot \sqrt{2 \cdot 2}}$$

$$= \frac{2 \cdot \left(-\frac{1}{2}\right) + 2 \cdot \frac{\sqrt{3}}{2}}{2 \cdot 2}$$

$$= \frac{\cancel{\sqrt{3}} - 1}{\cancel{4}}$$

$$= \frac{\sqrt{3} - 1}{4}$$
