4.)

a) $z = \sqrt{x^2 + y^2}$

$z_x = \frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}} \cdot 2x$

$z_y = \frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}} \cdot 2y$

$P_0 (3; 4; 5) \qquad \alpha = 45^\circ$

$$\frac{\partial z}{\partial \vec{a}} = z_x \cdot \cos\alpha + z_y \cdot \sin\alpha$$

$$\frac{\partial z}{\partial \vec{a}} = \frac{2x\cos\alpha + 2y\sin\alpha}{2\sqrt{x^2+y^2}} = \frac{x\cdot\cos\alpha + \sin\alpha \cdot y}{\sqrt{x^2+y^2}}$$

$$\frac{\partial z}{\partial \vec{a}} = \frac{3\cdot\cos(45^\circ) + 4\cdot\sin(45^\circ)}{\sqrt{3^2+4^2}}$$

$$\approx 0{,}99$$