**1.1.1 Die natürlichen Zahlen**

**a)**

$$5!=1⋅2⋅3⋅4⋅5=120$$

$$4!=1⋅2 ⋅3⋅4=\frac{5!}{5}=24$$

$$4!⋅5=\left(1⋅2⋅3⋅4\right)⋅5=5!$$

$$\frac{\left(n+1\right)!}{n!}=\frac{1⋅2⋅3…n⋅\left(n+1\right)}{n!}=n+1$$

$$\frac{\left(2n\right)!}{n!}=\frac{1⋅2⋅3⋅…⋅n⋅\left(n+1\right)⋅\left(n+2\right)⋅…⋅\left(2n\right)}{1⋅2⋅3⋅…⋅n}=\left(n+1\right)⋅\left(n+2\right)⋅…⋅\left(2n\right)$$

**b)**

$$\left(\begin{matrix}5\\3\end{matrix}\right)=\frac{5!}{3!⋅2!}=\frac{5⋅4⋅3⋅2⋅1}{1⋅2⋅3⋅1⋅2}=\frac{5⋅4⋅3}{1⋅2⋅3}=10$$

$$\left(\begin{matrix}5\\2\end{matrix}\right)=\frac{5!}{2!⋅3!}=\frac{5⋅4}{1⋅2}=10$$

$$\left(\begin{matrix}n\\k\end{matrix}\right)=\left(\begin{matrix}n\\n-k\end{matrix}\right)$$

$$\left(\begin{matrix}49\\6\end{matrix}\right)=\frac{49⋅48⋅47⋅46⋅45⋅44}{1⋅2⋅3⋅4⋅5⋅6}=13.983.816$$

$$\left(\begin{matrix}n\\0\end{matrix}\right)=1$$

**1.1.3.**

Beweis das $\sqrt{2}$ keine rationale Zahl ist.



**1.1.4**

**c)**

$$\left|x-3\right|<\left|x+1\right|$$



|  |  |  |
| --- | --- | --- |
| 1. Fall  | 2. Fall | 3. Fall |
| $$x\leq \left(-1\right)$$ | $$-1<x\leq 3$$ | $$x>3$$ |
| $$-\left(x-3\right)<-(x+1)$$$$-x+3<-x-1$$$$4<0 f.A.$$$$L\_{1}=∅$$ | $$-\left(x-3\right)<x+1$$$$-x+3<x+1$$$$1<x$$$$L\_{2}=(1,3]$$ | $$x-3<x+1$$$$-3<x+1$$$$-3<1 w.A.$$$$L\_{3}=(1,\infty )$$ |

$$L=L\_{1}∪L\_{2}∪L\_{3}$$

$$\overline{L=\left(1,\infty \right)}$$

Signum:

$$\left(x+1\right)^{2}=4$$

$$⇔\left|x+1\right|=2$$

$$⇔x+1=2∨x+1=-2$$

$$x=1∨x=-3$$

$$Für x\ne 0 ist\frac{x}{\left|x\right|}=\left\{\begin{matrix}\frac{x}{x}=1, wenn x>0\\-\frac{x}{x}=-1, wenn x<0\end{matrix}=sgn\left(x\right)\right.$$

**1.2.1**

**a)**

$$\left(a+2b\right)⋅\left(c-3a\right)=ac-3a^{2}+2bc-6ab$$

Aber:

$$a+2b⋅c-3a=-2a+2bc$$

$$18⋅22=\left(20+2\right)\left(20-2\right)=400-4=396$$

$$\frac{4a^{2}+12ab+9b^{2}}{4a^{2}-9b^{2}}=\frac{\left(2a+3b\right)^{2}}{\left(2a+3b\right)\left(2a-3b\right)}=\frac{2a+3b}{2a-3b}$$

$$\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\frac{\left(\sqrt{x}-\sqrt{y}\right)(\sqrt{x}-\sqrt{y})}{\left(\sqrt{x}+\sqrt{y}\right)(\sqrt{x}-\sqrt{y})}= \frac{\left(\sqrt{x}-\sqrt{y}\right)^{2}}{x-y}=\frac{x-2\sqrt{xy}+y}{x-y}$$

**b)**

$$\left(a+b\right)^{4}$$

$$=\left(\begin{matrix}4\\0\end{matrix}\right)b^{4}+\left(\begin{matrix}4\\1\end{matrix}\right)a^{1}b^{3}+\left(\begin{matrix}4\\2\end{matrix}\right)a^{2}b^{2}+\left(\begin{matrix}4\\3\end{matrix}\right)a^{3}b^{1}+\left(\begin{matrix}4\\4\end{matrix}\right)a^{4}$$

$$=b^{4}+4ab^{3}+6a^{2}b^{2}+4a^{3}b+a^{4}$$

**c)**

$$\frac{ax+ay}{x^{2}-y^{2}}=\frac{a\left(x+y\right)}{\left(x+y\right)\left(x-y\right)}=\frac{a}{x-y}$$

**1.2.2**

**a)**

$$2x^{2}-4x=6\rightarrow x^{2}-2x=3\rightarrow x^{2}-2x-3=0$$

$$\frac{2}{2}\pm \sqrt{\frac{4}{4}+3}$$

$$x\_{1}=-1$$

$$x\_{2}=3$$

$$3x^{2}-12x+15=0\rightarrow x^{2}-4x+5=0$$

$$+\frac{4}{2}\pm \sqrt{\frac{16}{4}-5}\rightarrow 2\pm \sqrt{4-5}\rightarrow 2\pm \sqrt{-1}\rightarrow Wurzel aus negativer Zahl für R nicht definiert.$$

$$4x^{2}+24x+36=0\rightarrow x^{2}+6x+9=0$$

$$-\frac{6}{2}\pm \sqrt{\frac{36}{4}-9}\rightarrow -3\pm \sqrt{9-9}$$

$$x\_{1,2}=-3$$

**b)**

$$x^{5}-9x^{3}-8x^{2}+72=0$$

$$\left(x^{3}-8\right)\left(x^{2}-9\right)=0$$

$$x\_{1}=2$$

$$x\_{2}=3$$

$$x\_{3}=-3$$

**1.3**

$$i\rightarrow Mathematik$$

$$j\rightarrow Elektrotechnik$$

$$i=\sqrt{-1}\rightarrow imaginäre Einheit⇒\sqrt{-\left|a\right|}=\sqrt{\left|a\right|}i$$

**1.3.1.**

**a)**



**b)**

$$\rightarrow j=\sqrt{-1}\rightarrow \sqrt{-1}⋅\sqrt{-1}=-1=j⋅j=j^{2}$$

Beispiele:

1. $\left(3-2j\right)+\left(4+7j\right)=\left(3+4\right)+\left(-2+7\right)j=7+5j$
2. $\left(3-2j\right)-\left(4+7j\right)=-1-9j$
3. $\left(3-2j\right)\left(4+7j\right)=3⋅4+3⋅7j-2j⋅4-2j⋅7j=12+21j-8j+14=26+13j$

**c)**

$$\left(a+bj\right)\left(a-bj\right)=a^{2}+abj-abj-\left(bj\right)^{2}=a^{2}-b^{2}j^{2}=a^{2}-b^{2}⋅\left(-1\right)=a^{2}+b^{2}$$

$$\left(a+bj\right)\left(a-bj\right)=a^{2}+b^{2}$$

**d)**

$$z=4-3j⇒\left|z\right|=\sqrt{4^{2}+\left(-3\right)^{2}}=\sqrt{25}=5$$

$$z=-5+3j=\left|z\right|=\sqrt{\left(-5\right)^{2}+3^{2}}=\sqrt{34}$$

**e)**

$$\frac{2+3j}{3-4j}=\frac{\left(2+3j\right)(3+4j)}{\left(3-4j\right)(3+4j)}=\frac{6+8j+9j-12}{9+16}=\frac{-6+17j}{25}=-\frac{6}{25}+\frac{17}{25}j$$

**1.3.2**

|  |  |
| --- | --- |
| C:\Users\JD\Pictures\Eigene Scans\Scan_Pic0010.jpg | $$\left|z\right|=\sqrt{a^{2}+b^{2}}=z⋅\overbar{z}$$$$\left|z\right|⋅(\cos(a)+j⋅\sin(α))$$ |

**a)**



$$z=3+4j⇒\left|z\right|=5 und \tan(α)=\frac{4}{3}⇒α=0,93≜53°$$

$$z=3-4j⇒\left|z\right|=5 und \tan(α)=-\frac{4}{3}⇒da im 2.Quadrant α=180°-53°=127°$$

$$z=-3-4j⇒\left|z\right|=5 und \tan(α)=\frac{4}{3}⇒da im 3.Quadrant α=180°+53°=233°$$

$$z=-3+4j⇒\left|z\right|=5 und \tan(α)=-\frac{4}{3}⇒ α=-53° oder 360°-53°=307°$$

**b)**

Beispiele:

Man berechne die 6. Potenz von $z=2-2j$.

$z^{k}=$ $\left|z\right|^{k}⋅\left[\cos(\left(kα\right)+j⋅\sin(\left(kα\right)))\right]$

$$z=2-2j$$

$$\tan(α=-\frac{2}{2}=-1)$$

$$α=-45°=315°$$

$$\left|z\right|=\sqrt{8}=2\sqrt{2}$$

$$z^{6}=\left(2\sqrt{2}\right)^{6}⋅\left[\cos(\left(6⋅315\right)+j⋅\sin(\left(6⋅315\right)))\right]$$

$$z^{6}=512⋅\left(0+1j\right)$$

$$z^{6}=512⋅0+512⋅1j$$

$$z^{6}=512j$$

**1.3.3.**

Beispiele:

$$\sqrt[n]{z}$$

$$k=0,1,2,…,n-1$$

$$w\_{k}=\sqrt[n]{\left|z\right|}⋅\left[\cos(\left(\frac{α}{n}+k⋅\frac{2π}{n}\right)+j⋅\sin(\left(\frac{α}{n}+k⋅\frac{2π}{n}\right)))\right]$$

$$\sqrt[4]{3+4j}$$

$$\left|z\right|=\sqrt{3^{2}+4^{2}}=5$$

$$w\_{0}=5⋅\left[\cos(\left(\frac{\arctan(\left(\frac{4}{3}\right))}{4}+0⋅\frac{2π}{4}\right)+j⋅\sin(\left(\frac{arctan\left(\frac{4}{3}\right)}{4}+0⋅\frac{2π}{4}\right)))\right]=1,46+0,34j$$

$$w\_{1}=-0,34+1,46j$$

$$w\_{2}=-1,46-0,34j$$

$$w\_{3}=0,34-1,46j$$

